

# A Global System for Matching 3-D Range Data Objects

Nabih N. Abdelmalek  
Division of Electrical Engineering  
National Research Council  
Ottawa, Ontario, Canada K1A 0R8

## Abstract:

Experimental results for matching 3-D range data objects based on a global approach is here presented. The obtained invariant parameters used for matching purposes are functions of the geometrical dimensions of the object, the second and fourth order central moments of both the object depth and the contour of its base silhouette.

A separate set of invariant parameters are calculated and stored into the matching library for each stable position for each used object. The matching system is tested on a number of real objects and was found to be fast and robust.

## 1. INTRODUCTION

Recognition and matching of three dimensional (3-D) objects from camera views is an important problem in image recognition. It has wide applications for robotic vision systems.

The basic idea in 3-D object recognition and matching is to describe the object by a set of parameters (primitives) and to match these parameters with sets of stored parameters for model objects. Such parameters are to be invariant under object translation, plane rotation and if possible under change of scale.

Many of the 2-D object recognition methods are extended to the recognition of 3-D objects, such as the Fourier descriptors [1,2] and the method of invariant moments [3-10]. There are also classical global methods based on the object shape and size [11-14].

Classical approaches for recognition and matching of images are not new. See for example Bacus and Gose [11] for medical application using binary images and Darling and Joseph [12] for scene analysis using gray level images. Classical statistical approaches are also used with other techniques for texture recognition. See for example Hawkins [13] and Gonzalez and Wintz [14].

In the present work, we apply a classical global approach to the problem of matching 3-D range data objects. The obtained invariant parameters used for matching purposes are functions of the shape and size of the object. We report experimental results concerning the reliability and robustness of this approach.

### 1.1 The invariant matching parameters

For matching purposes we calculate the following 10 invariant parameters.

- (1) The volume of the object,
- (2) The maximum height of the object,
- (3) The object base area or its mean height,
- (4) The standard deviation of the object surface depth,
- (5) The standard deviation of the  $x$  and  $y$  coordinates [8] of the object surface,
- (6) The (relative) maximum or the (relative) minimum moment of inertia of the object surface about axes through the point  $(x_{mean}, y_{mean}, 0)$ . By relative maximum or relative minimum, we mean the maximum or minimum moment of inertia divided by the volume of the object. See section 3 below.
- (7) The radius of the sphere of center  $(x_{mean}, y_{mean}, 0)$  which fits the object surface in a least squares sense.  $(x_{mean}, y_{mean})$  are the  $(x, y)$ -coordinates of the object surface centroid.
- (8) The standard deviation of the  $x$  and  $y$  coordinates of the contour of the base silhouette,
- (9) The relative maximum or the relative minimum moment of inertia of the contour of the object base silhouette, about axes through the contour centroid. Again, here by relative we mean the maximum or minimum moment of inertia of the contour of the base silhouette divided by the number of points of the contour of the base silhouette. See section 3 below.
- (10) The radius of the circle of center  $(x_{mean}^c, y_{mean}^c)$  which fits the contour of the base silhouette in a least squares sense. Again,  $(x_{mean}^c, y_{mean}^c)$  are the  $(x, y)$ -coordinates of the centroid of the contour of the object base silhouette.

Parameters 1-3 are functions of the object shape and size. As we see in section 3 below, parameters 4-6 and 8-9 are functions of different second order moments of the object surface and the contour of its base silhouette. Also parameters 7 and 10 are functions the second and fourth order central moments of the object coordinates and those of the object base silhouette respectively.

The above given matching parameters are invariant under translation and plane rotation but not under a change of scale. Nevertheless, we here mention an important point. That the data obtained from a range camera are already calibrated as we get the (x,y,z) coordinates of the object surface. Therefore we need no more worry about the position of the camera from the scanned object, which causes the change of scale.

The angle that the axis of maximum (minimum) moment of inertia of the object surface makes with the I- axis is easily calculated and may be used for picking purposes in a robot vision system. Also the angle that the axis of maximum (minimum) moment of inertia of the base silhouette contour makes with the I- axis may be used as an alternative for picking purposes.

A separate set of invariant parameters are calculated and stored for the matching library for each stable position for each used object.

## 2. DESCRIPTION OF THE SYSTEM

### 2.1 Data forms

Recently a new laser range image camera was designed in the Division of Electrical Engineering, of the National Research Council of Canada [15-17]. It provides a means to obtain a very large field of view without compromising on resolution. Besides, it gives a large increase in speed of measurement and the shadow effects are reduced considerably.

The range data from the camera is given in a Real\*4 (R\*4) string form; {x(1), y(1), z(1), x(2), y(2), z(2), ..., x(k), y(k), z(k)}, which are respectively the x-, y- and z-coordinates of successive points of the object. We denote this by the x-y-z- string vector presentation of the object surface. In this data, in general, the x-coordinates are at equal intervals but the y-coordinates are not at equal intervals or vice versa.

This data is then interpolated and presented with the background in the form of an R\*4 256x256 matrix. The matrix (i,j) element is the z-coordinate value at point (i,j), i.e. at the (x,y) coordinates, which are integers. We denote this by the matrix presentation of the object.

### 2.2 Reducing the processing time

To reduce the processing time of the matching system to about one quarter of its amount, when a 256x256 range object matrix is read, the object matrix is reduced to a 128x128 one. This is done by taking every second matrix element, every second row. The invariant parameters of the system are calculated from the obtained 128x128 matrix.

### 2.3 Further reducing the processing time

Again to further reduce the processing time, the following files are obtained and utilised.

The 3 R\*4 x-, y- and z- vectors respectively for the i, j and z coordinates of the surface at hand. That is, given an object surface in a matrix form, we separate the object from the background and obtain the object surface in a set of 3 string forms, which we denote by the x-, y- and z- string vectors of the object. Each string is of length k, the number of the surface points of the object.

A 128x128 object matrix contains 16384 elements. Yet the number of nonzero matrix elements k, may range from about 2000 to 6000 elements. We do our computation using the 3 k-string vectors of the object surface, with a substantial reduction of the processing time. The processing time is reduced by the ratio k to the total matrix elements, 16384.

### 2.4 The contour of the object base silhouette

By the contour of the object base silhouette, we mean the outside and also the inside contours of the two dimensional object base. If the 3-D object is hollow, the base will show holes inside its boundaries. The inside contours are the contours of these holes.

These contours are obtained by the following simple technique. The object matrix is scanned column-wise (row-wise). Consider every non-zero element of this matrix. Each of the 8 neighbouring elements to this non-zero element as a center is examined. If any of these 8 elements is a zero element, the center element is considered a boundary or a contour point of the object base silhouette. The elements of the base contours are each given a character 1 and the rest of the base elements each is given the blank character ' '. Figure 3 shows the contour of the base silhouette for 4 orientations of the range object number 3.

We notice in figure 3 that the number of contour elements of the object base silhouette is not the same for the 4 orientations of the object. Similarly as mentioned before, we notice from table 1 below that the calculated volume of the object is slightly different for the four views of each object. This point was a factor in our choice of the statistical technique in which the invariant matching parameters are not sensitive to the mentioned variations of the object volume and number of points on the contour of the base silhouette.

## 3. CALCULATING THE INVARIANT PARAMETERS

The significance of the invariant parameters is obvious as calculated in this section.

(1) The object volume is none other than the sum of the k elements in the z-string vector of the object surface.

(2) The maximum height of the object, is obviously an invariant parameter and is easy to obtain.

(3) The object base area, is clearly the length

'k' of any of the 3 string vectors of the object surface. The base area thus has an integer value.

The coordinates of the surface centroid  $(x_{mean}, y_{mean}, z_{mean})$  are given respectively by

$$\begin{aligned} x_{mean} &= \sum_{i=1}^k x_i/k \\ y_{mean} &= \sum_{i=1}^k y_i/k \\ z_{mean} &= \sum_{i=1}^k z_i/k \end{aligned} \quad (1)$$

In (1),  $z_{mean}$  is the mean or average height of the object, which is obviously an invariant parameter.

(4) The standard deviation  $\sigma_z$  of the object's surface z-coordinates is given by

$$\sigma_z^2 = \sum_{i=1}^k (z_i - z_{mean})^2/k \quad (2)$$

The central second moments of the object surface are given by

$$\begin{aligned} I_{xx} &= \sum_{i=1}^k z_i (y_i - y_{mean})^2 \\ I_{yy} &= \sum_{i=1}^k z_i (x_i - x_{mean})^2 \\ I_{xy} &= \sum_{i=1}^k z_i (x_i - x_{mean})(y_i - y_{mean}) \end{aligned} \quad (3)$$

(5) The standard deviation of x-y coordinates of the object surface is a parameter used by Alt [8] is given by

$$\begin{aligned} \sigma_{x-y}^2 &= \sigma_x^2 + \sigma_y^2 \\ \text{where} \quad \sigma_x^2 &= I_{yy}/k \\ \sigma_y^2 &= I_{xx}/k \end{aligned} \quad (4)$$

This parameter  $\sigma_{x-y}$  is proportional to the square root of first second-order invariant moment parameter  $\phi^{(1)}$  ([7], p. 697).

(6) The maximum (minimum) central moments of inertia of the object surface about axes through the point  $(x_{mean}, y_{mean}, 0)$ , is a measure of the elongation of the object along the proper axis. These are given by the following formulas [18].

$$\begin{aligned} 2 I_{max} &= (I_{xx} + I_{yy}) + \text{sqrt}[(I_{xx} - I_{yy})^2 + 4I_{xy}^2] \\ 2 I_{min} &= (I_{xx} + I_{yy}) - \text{sqrt}[(I_{xx} - I_{yy})^2 + 4I_{xy}^2] \end{aligned} \quad (5a)$$

and the direction of the principal axes is given by

$$\tan 2\theta = 2I_{xy}/(I_{yy} - I_{xx}) \quad (5b)$$

If  $I_{xy}=0$ ,  $2\theta=0$  or  $180$  and if  $I_{xx}=I_{yy}$ ,  $\theta=\pi/4$ .

It is found that as the object is translated and/or rotated about its z axis, the shadow effect results in some variations in the values of the calculated parameters 1-6. Hence we divide the maximum (minimum) central moment of inertia by the volume of the object. We get the relative maximum (minimum) central moment of inertia. Which, similar to the standard deviations (parameters 4 and 5) is an invariant parameter.

Grosky and Jain [19] suggested fitting the object surface in a least squares sense by an elliptic paraboloid. The intersection of this elliptic paraboloid with the plane  $z=0$  is an ellipse, whose eccentricity is supposed to be invariant under translation, plane rotation and

size. We have experimented with such surfaces and found out that the values of the parameters of the ellipse are sensitive to the shadow effects and the calculated values for the same object with different orientations vary substantially. We also found out that for some objects, the fitting surface is a hyperbolic paraboloid and not an elliptic paraboloid.

Instead, in the present work we fit the object surface in the least squares sense with a sphere whose center is the point  $(x_{mean}, y_{mean}, 0)$ .

The equation of the fitting sphere may be given by

$$a_1(X^2 + Y^2 + Z^2) = 1 \quad (6a)$$

where

$$\begin{aligned} X &= x - x_{mean} \\ Y &= y - y_{mean} \end{aligned} \quad (6b)$$

and again  $(x_{mean}, y_{mean})$  are the  $(x, y)$ -coordinates of the surface centroid.

Substituting the k coordinates x, y and z of the 3 k-string vectors into eqs. (6a,b) we get the system of linear equations

$$\begin{bmatrix} X_1^2 + Y_1^2 + Z_1^2 \\ X_2^2 + Y_2^2 + Z_2^2 \\ \vdots \\ X_k^2 + Y_k^2 + Z_k^2 \end{bmatrix} a_1 = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \quad (7a)$$

In matrix-vector form, eq. (7a) may be written as

$$Ca_1 = f \quad (7b)$$

where C is the coefficient matrix in (7a) which is in fact a vector of length k and f is the r.h.s. k-vector in (7a), each element of which is 1. In (7a,b), 'a<sub>1</sub>' is the unknown parameter. The radius of the fitting sphere =  $1/\text{sqrt}(a_1)$ .

The least squares solution parameter 'a<sub>1</sub>' is the solution of the normal equation

$$C^T C a_1 = C^T f \quad (8)$$

where  $C^T$  the transpose of vector C.

Each of  $C^T C$  and  $C^T f$  in (8) is a scalar. They have the values

$$C^T C = \sum_{i=1}^k (X_i^4 + Y_i^4 + Z_i^4 + 2X_i^2 Y_i^2 + 2X_i^2 Z_i^2 + 2Y_i^2 Z_i^2) \quad (9a)$$

$$\text{and} \quad C^T f = \sum_{i=1}^k (X_i^2 + Y_i^2 + Z_i^2) \quad (9b)$$

Expression (9a) is a fourth order moment while that of (9b) is a second order moment expression.

(8) The formula for the standard deviation of the x-y coordinates of the contour of the base silhouette is similarly obtained from formula similar to (4).

(9) The maximum and the minimum central moments of inertia of the contour of the base silhouette are calculated from the same formulas (3), with the  $z_i$  replaced by 1. The summation in (3) would be taken over the number of points of the base contour silhouette.

(10) The radius of the circle which fits the contour of the base silhouette in a least squares sense is easily calculated. This is done in an analogous way to calculating the radius of the fitting sphere in (7) above. Again the radius of this circle is a function of the second and fourth order moments of the contour of the object base silhouette.

#### 4. EXPERIMENTAL RESULTS

We experimented with 10 given industrial-like objects. Some of them resemble each other in shape and size. Figure 1 shows one view of each of the 10 given objects. Each of the 10 objects is translated and rotated an arbitrary angle about the object  $z$  axis 4 times. Thus in all we have data for 40 object views.

As soon as the matrix for the given object is read it is reduced to an  $128 \times 128$  matrix.

Table 1 shows the calculated invariant parameters (1) - (10) for the 10 objects. The l.h.s. numbers (1), (2), ..., refer to the object number.

Table 1 shows the clustering of the parameter values for the same object with different orientations. The results indicate that the smallest percentage variations in the parameters exist for almost all the parameters except parameters 6 and 9. We denote these parameters by set 1. This makes set 1 the most reliable parameters for the matching purposes. More percentage variations occur for parameters 6 and 9, in the maximum moments of inertia of the object surface and the maximum moments of inertia of the contour of the base silhouette, which we denote by set 2.

Nonetheless, we have given each parameter the same weight. Matching is done by counting the largest number of times the object parameters are closest, percentage wise, to those of the model.

The experimental results show that this matching system is fast and robust.

#### ACKNOWLEDGEMENTS

The author had interesting discussions with Nestor Burtnyk, Shadia Elgazzar and Mark Rioux. Robert Kadamani made this system an interactive one on the IIS display. Kenneth Evans, Arthur Binch and Michael Duggan helped with the IIS display, the Vax system and the Camera. I am thankful to all of them. A special thanks is to Luc Cournoyer for providing us with the Range Data.

#### REFERENCES

1. C.T. Zahn and R.Z. Roskies, Fourier descriptors for plane closed curves, IEEE Trans. on Computers, vol. C-21, pp. 269-281, 1983.
2. C.W. Richard and H. Hemami, Identification of three - dimensional objects using Fourier descriptors of the boundary curve, IEEE Trans. on Pattern Analysis and Machine Intelligence, vol. PAMI-2, pp. 127-136, 1980.
3. M.-K. Hu, Visual pattern recognition by moment invariants, IRE Trans. on Information Theory, vol. IT-8, pp. 179-187, 1962.
4. S.A. Dudani, K.F. Breeding and R.B. McGee, Aircraft identification by moment invariants, IEEE Trans. on Computers, vol. C-26(1), pp. 39-45, 1977.
5. R.Y. Wong and E.L. Hall, Scene matching with invariant moments, Computer Graphics and Image Processing, vol. 8, pp. 16-24, 1978.
6. E.L. Hall, Computer Image Processing and Recognition, Academic Press, 1979.
7. S. Maitra, Moment invariants, Proceedings of the IEEE, vol. 67, pp. 697-699, 1979.
8. F.L. Alt, Digital pattern recognition by moments, ACM Journal, vol. 9, pp. 240-258, 1962.
9. T.C. Hsia, A note on invariant moments in image processing, IEEE Trans. on Systems, Man and Cybernetics, vol. SMC-11(12), pp. 831-834, 1981.
10. G.L. Cash and M. Hatamian, Optical character recognition by the method of moments, Computer Vision, Graphics and Image Processing, vol. 39, pp. 291-310, 1987.
11. J.W. Bacus and E.E. Gose, Leukocyte pattern recognition, IEEE Trans. on System, Man and Cybernetics, vol. SMC-2, pp. 513-526, 1972.
12. E.M. Darling and R.D. Joseph, Pattern recognition from satellite altitudes, IEEE Trans. SSC, vol. SSC-4, pp. 38-47, 1968.
13. J.K. Hawkins, Textural properties for pattern recognition, in "Picture Processing and Psychopictorics", B.S. Lipkin and A. Rosenfeld (Eds.), Academic Press, New York, 1969, pp. 347-370.
14. R.C. Gonzalez and P. Wintz, Digital Image Processing, Addison-Wesley, Reading, Mass., 1987.
15. M. Rioux, Laser range finder on synchronized scanners, Applied Optics, vol. 23, pp. 3837-3844, 1984.
16. F. Bumbaca, F. Blais and M. Rioux, Real-time correction of three-dimensional nonlinearities for a laser range finder, Optical Engineering, vol. 25(4), pp. 561-565, 1986.

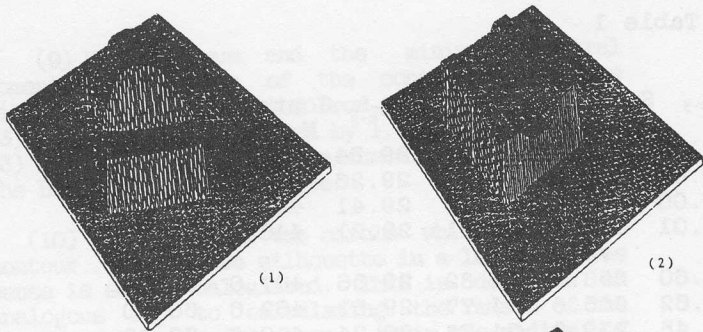
Table 1

Object Volume	$z_{max}$	$z_{mean}$	$\sigma_z$	$\sigma_{x-y}$	Surface M.O.I.	Sphere radius	$\sigma_{x-y}^c$	Contour M.O.I.	Circle radius
(1) 81308.	53.97	30.56	12.34	18.99	196.2	42.98	29.34	456.2	30.15
81841.	53.84	30.21	12.66	19.01	192.5	42.87	29.26	447.4	30.02
82830.	53.83	30.71	12.48	19.00	184.9	42.98	29.41	458.6	30.19
82386.	53.97	30.43	12.56	19.01	191.8	42.91	29.21	446.0	29.95
(2) 64632.	29.38	23.74	5.41	21.60	258.9	34.82	29.56	467.0	30.39
64798.	29.91	23.53	5.72	21.62	255.8	34.77	29.51	462.6	30.30
63674.	29.28	23.85	5.32	21.45	239.8	34.78	29.24	449.6	30.10
64733.	29.79	23.47	5.83	21.61	252.7	34.76	29.50	457.5	30.31
(3) 18515.	24.81	21.83	5.37	15.50	132.2	28.95	19.44	214.3	22.82
18730.	24.86	21.88	5.29	15.47	132.5	28.88	19.10	200.0	22.52
18853.	24.49	21.45	5.70	15.47	133.9	28.64	19.13	211.8	22.48
18733.	24.30	21.86	5.34	15.44	131.6	28.85	19.18	200.2	22.54
(4) 14474.	17.41	11.38	4.41	15.37	131.0	21.87	20.10	213.2	20.72
14650.	17.75	11.33	4.41	15.43	133.8	21.86	19.95	208.1	20.54
14855.	17.72	11.41	4.33	15.47	137.9	21.88	20.20	215.3	20.75
14761.	17.41	11.36	4.37	15.47	134.6	21.87	20.02	209.4	20.58
(5) 48169.	27.49	21.71	5.12	20.12	300.1	32.01	27.42	510.0	29.26
41350.	27.53	21.75	4.90	19.06	280.5	31.20	25.46	465.7	27.69
48775.	27.60	22.03	4.44	20.45	316.3	32.38	27.69	549.0	30.09
48604.	27.50	21.81	4.83	20.28	307.3	32.16	27.40	524.3	29.48
(6) 51482.	32.97	23.96	5.83	19.83	297.3	33.70	27.02	501.3	29.03
52028.	32.99	23.74	6.21	20.06	308.3	33.83	27.20	519.3	29.36
52131.	32.95	23.99	5.72	20.23	317.2	33.99	27.57	551.6	30.04
52029.	33.22	23.62	6.24	20.04	307.0	33.76	27.28	520.7	29.39
(7) 32026.	16.92	11.19	5.38	25.19	477.5	31.47	30.69	601.2	33.26
31780.	17.12	11.20	5.37	25.15	475.3	31.40	30.41	580.2	33.32
31990.	17.05	11.15	5.38	25.37	493.2	31.64	30.58	605.0	33.55
32033.	17.08	11.12	5.37	25.23	479.3	31.44	30.76	601.8	33.31
(8) 37990.	29.90	14.63	7.45	18.71	197.1	29.68	29.27	573.3	31.51
38020.	29.65	14.60	7.44	18.75	201.8	29.79	29.39	583.8	31.98
37932.	29.82	14.66	7.47	18.67	199.6	29.68	29.24	576.5	31.69
38015.	29.63	14.60	7.46	18.67	195.9	29.55	29.27	569.4	31.81
(9) 39458.	26.96	14.19	6.28	21.31	339.0	32.59	31.64	726.6	34.98
39145.	26.98	14.22	6.24	21.10	328.9	32.14	31.34	703.2	34.59
38614.	26.90	14.31	6.22	20.99	325.9	32.16	31.45	728.5	34.96
39143.	26.92	14.19	6.28	21.09	326.7	32.19	31.43	708.4	34.54
(10) 63904.	26.39	17.22	5.84	25.87	503.0	36.30	35.69	897.5	38.57
63718.	26.35	17.08	5.91	25.74	493.1	36.04	35.65	885.3	38.33
63191.	26.20	16.97	5.82	25.81	498.3	35.99	35.67	885.6	38.36
63362.	26.23	17.17	5.61	25.98	510.6	36.30	35.92	914.8	38.85

17. F. R. Livingstone and M. Rioux, Optical techniques for industrial inspection, Proc. SPIE-The International Society for Optical Engineering, vol. 665, pp. 188-194, 1986.

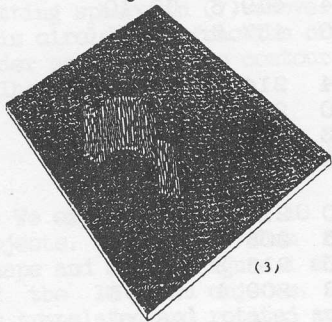
18. A. Rosenfeld and A.C. Kak, Digital Picture Processing, Academic Press, New York, 1976.

19. W.I. Grosky and R. Jain, Pyramid-based approach to segmentation applied to region matching, IEEE Trans. on Pattern Analysis and Machine Intelligence, vol. PAMI-8, pp. 639-650, 1986.

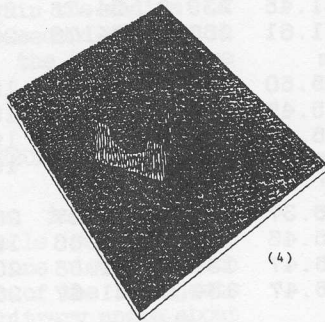


(1)

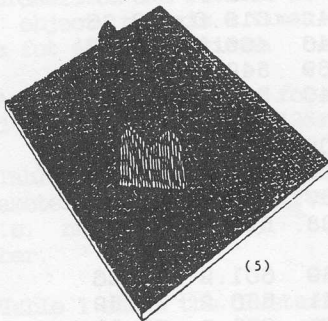
(2)



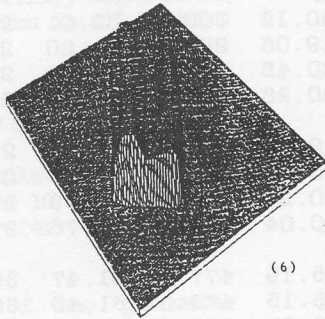
(3)



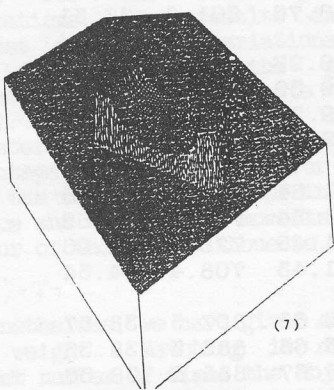
(4)



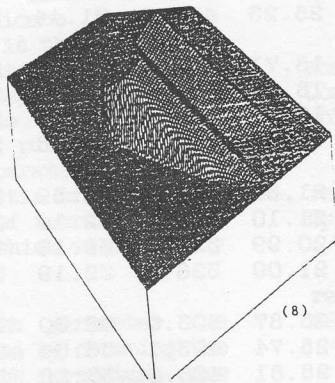
(5)



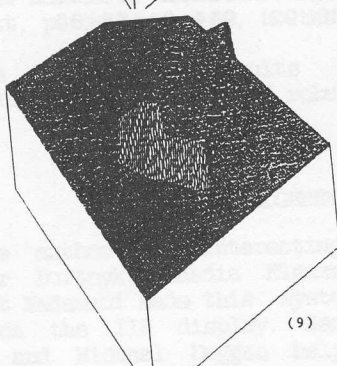
(6)



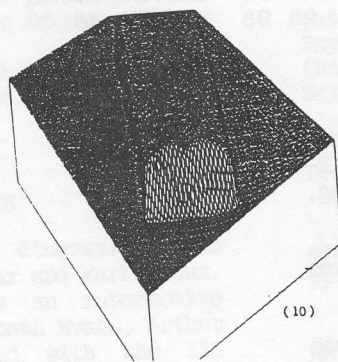
(7)



(8)



(9)



(10)

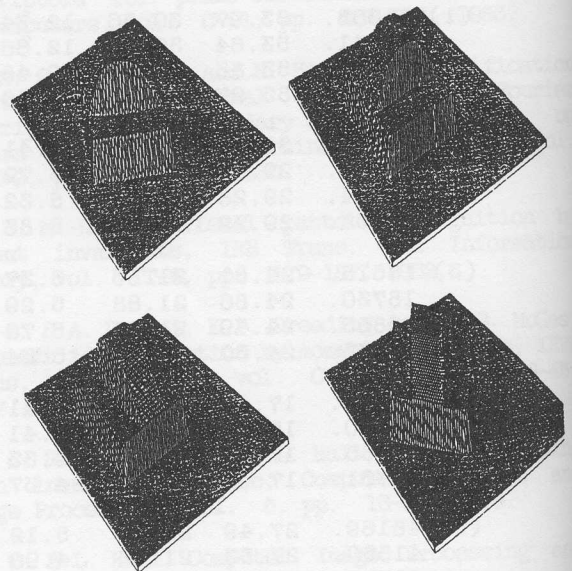


Figure 2. The four views of the object 1.

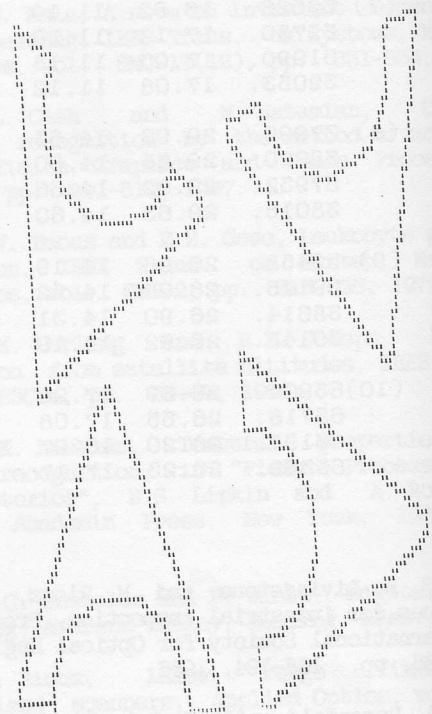


Figure 3. The contour of the base silhouette for the four views of object 3.

Figure 1. One view of each of the ten range objects.