# Deformable Primitives in Axial Representations of Shapes †

**R. Laganière, P. Cohen**

*Department of Electrical Engineering*
*Ecole Polytechnique*
*P.O.B. 6079 Station A*
*Montréal, Qué.*
*Canada H3C 3A7*

**RESUMÉ :** Plusieurs critères ont été jusqu'ici proposés pour l'évaluation de la qualité d'une représentation de formes. En particulier, la richesse du support local et l'aptitude à résumer l'information (Brady[1983]) sont considérés comme des conditions primordiales dans l'élaboration d'un mode de représentation efficace. Cet article présente un mode de représentation régionale de formes bidimensionnelles, qui tout en satisfaisant ces critères mène à une description extrêmement concise. Partant de la représentation axiale d'une forme, la procédure proposée définit un ensemble de primitives déformables choisies de façon à améliorer la consistance locale de chaque primitive avec la forme étudiée, tout en maximisant sa contribution spatiale à l'intérieur de la forme.

**ABSTRACT:** Several important criteria have been suggested in the literature for assessing the adequacy of a representation of shape. In particular, rich local support and subsumption (Brady[1983]) are considered to be key requirements for an efficient representation. This paper presents a region-based representation of two- dimensional shapes which tries to satisfy these requirements and leads to extremely concise descriptions. Starting from an axial representation of the shape under consideration, the proposed procedure defines new locally deformable primitives in such a way as to improve the local consistency between the shape and the primitives, while maximizing the spatial contribution of each primitive to the inside of the shape.

**KEYWORDS:** Region-based Representation, Axial Representation, Primitive Deformation, Medial Axis Transform.

## 1. Introduction.

The search for rich representations of the information available in images constitutes one of the main problems in computer vision. Any process of image analysis has two associate representations: one corresponding to its input and the other corresponding to its output (or equivalently to the input of the subsequent processing stage). A crucial dependency thus exists between the performance of a computer vision task and the qualities of the representations on which it is based. Several criteria have been proposed, by Marr[1982], Brady[1983] and others, for assessing the adequacy of a representation:

- a representation should be efficiently computable (accessibility).
- it should represent information at a variety of scales (scope and sensitivity).
- it should preserve information and be locally computable (rich local support).
- whenever possible, local descriptions should give rise to more global ones (subsumption).

In the particular case of two-dimensional shapes, the bibliography is extensive (see Ballard[1982], Davis[1986]) and reveals two classes of representations: those that refer to the one-dimensional contours of objects and those that describe the two-dimensional region occupied by an object.

### 1.1 Contour-based representations.

Although shape is intrinsically a two-dimensional notion, contour-based description are sometimes preferred owing to their simplicity and their robustness to partial occlusion. The various polygonal approximation methods (Ramer[1977]) are examples of this type of representation. Although, such methods are widely used, the resulting descriptions often suffer from lack of stability and do not easily lend themselves to the computation of shape characteristics such as surface, main axes, etc. More elaborate contour-based representations based on generalized Hough transform (Ballard[1981]), on curvature primitives (Asada[1986], Mokhtarian[1986]), or on autoregressive models (Dubois[1986]) produce better descriptions in terms of the above criteria, but involve more computation.

### 1.2 Region-based representations.

Region-based representations, for the most part, involve covering the shape under consideration with a series of instances of a simple standard-shaped primitive. The covering strategy may involve either decomposition or construction.

The decomposition approach attempts to subdivide the original shape into a minimal set of non-overlapping simpler elements which, when combined, form an approximation of the original shape. The main difficulty with this approach lies in the selection of a primitive element which lead to concise and yet precise representations of shapes under consideration. Representation through decomposition into quadtree (Grotsky[1983]) is based on this idea.

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One possible primitive in this case is a square shaped element which eventually goes down the pixel level in order to describe significant details. Other techniques (Guerra[1982], Asano[1983], Ferrari[1984]), although applied mainly to polygonal figures, can be used to decompose shapes into simple polygons, most often convex.

In the constructive approach, the original shape is considered to be the union of overlapping simple components. Allowing the components to overlap reduces the difficulty of choosing a proper primitive element; however, the main problem lies now in the selection of a covering strategy which limits the complexity of the description. One of the best known representations of this type, the medial axis transform first introduced by Blum([1973], [1978]), basically involves the extraction of local symmetries of shapes. Its principle formed the basis of numerous later works on axial representations (Lee[1982], Samet[1983], Brady[1983], Nackman[1985], Wu[1986]). Axial representations essentially proceed by covering a shape with overlapping instances of a primitive that displays some kind of symmetry. Optimal coverage is obtained by considering only instances of maximal size that touch the boundary of the shape in at least two non-contiguous points. The description of the shape reduces then to the set of positions and sizes of all instances. The geometrical and topological characteristics of an axial representation largely depend upon the choice of the primitive's morphology. A description and a comparison of different representations based on this principle is provided by Rosenfeld[1986].

One drawback common to these axial representations, to varying degrees, is that they do not consistently produce axes whose length is proportional to the size of the corresponding details of the shape. This disadvantage leads to particularly lengthy descriptions in situations where the morphology of the detail to be represented differs substantially from that of the primitive. This poor performance may be observed, for example on polygonal figures, in the case of a disk-shaped primitive. A better result could probably be obtained if the primitive's morphology was locally modified so as to increase its similarity with the shape under consideration.

A second shortcoming of axial representations stems from the fact that, by virtue of their definition, these representations do not restrict the amount of overlap between consecutive components. Consequently, the great majority of components present a high degree of mutual overlap and thus contribute a negligible amount to the spatial occupation of the shape. Here again a better performance (in terms of conciseness of the description) could be obtained if the morphology of the primitive was modified locally, in such a manner as to enclose the union of successive instances of the original primitive presenting a high degree of mutual overlap.

Both previous remarks suggest, as a significant improvement of axial representations, the definition of a deformable primitive which would, on the one hand, try to replicate the local aspect of the shape under consideration (rich local support) and, on the other hand, maximize the spatial contribution of each instance of the primitive (subsumption). Obviously, the description of each instance of the primitive will become lengthier since one must specify not only its position and global size but also the characteristics of the applied deformation. However, the substantial decrease in the number of instances should more than compensate for this variation, thus producing a more concise description of the shape under consideration. This idea of deformation of primitives has already been proposed by several authors. In particular, Barr[1981] and Pentland[1986] discuss the value of deformations in the development of concise descriptions of shape. Although, both authors propose superquadrics as primitives, owing to their great flexibility, neither one of them has come up with a procedure for the computation of stable representations in terms of these primitives. A somewhat similar concept is presented by Rearick[1987]. However the type of primitive defined by the author does not, in general, procure enough flexibility to give rise to a precise representation.

The aim of this paper is to define a region-based representation of shapes, which satisfies criteria of rich local support and subsumption, using deformable primitives. It also presents as an application of this concept, a generalization of the medial axis transform (MAT). First, the MAT of the shape under consideration is established. Then a sequential search identifies maximal components in the form of symmetrically deformed disks. The type and degree of disk deformation corresponding to each component, as well as its spatial extent, are computed locally from the MAT description. This gives rise to a very concise description in which whole sections of the MAT are replaced by a single component occupying the same portion of the shape.

The following sections detail the proposed procedure, and discuss some of the results obtained with this type of representation.

## 2. Deformable primitives in axial representations.

### 2.1 The general approach.

According to the previous observations, the elaboration of the new type of representation requires, first, the choice of a specific axial representation and its corresponding basic primitive. A careful examination of shape descriptions produced by such an axial representation reveals the presence of two types of components (i.e. primitive instances). First, some components, owing to their particular locations, contribute a substantial proportion of their surface to the domain of the shape; we will call them hinge components. Limiting a shape's description to its hinge components will usually produce a coarse but informative approximation of the shape. Components of a second type, usually present in far greater number, account for the local details of the shape. Their individual spatial contributions are small and their presence is mainly due to the rigidity of the representation; we will call them secondary components. It thus seems appealing to try to detect the presence of hinge components by means of an appropriate exploration of the axial description. Then as a second step, one could attempt to eliminate the explicit presence of secondary components and replace their contribution by a suitable deformation of the hinge components.

The efficiency of such an approach relies on the solution of three main problems:

- The choice of a satisfactory axial representation.
- The reliable detection of hinge components.
- The definition of a proper class of primitive deformations.

The following sections present a particular application of the preceeding notions in the context of the medial axis transform representation. Particular choices are made concerning the solutions to the three preceeding problems. These choices are by no means unique, but rather serve as an illustration of the potential of the proposed approach.

### 2.2 Computation of the medial axis transform (MAT).

The choice of a particular type of MAT, in terms of its primitive's form and its definition of distance, is likely to influence the ultimate efficiency of the representation by making explicit particular symmetries of the shape under study (Brady[1983]). Among all alternatives (Rosenfeld[1968] and [1982], Danielsson[1980]), the MAT that uses disks as primitives and euclidian distances, although requiring more computation, appears to be preferable, owing to its invariance under shape rotation.

Computation of the MAT is highly sensitive to contour noise, as evidenced by the frequent presence of long axes corresponding to tiny contour details. Numerous methods have been developed to reduce this effect. For instance, one can pre-process the shape prior to computing the MAT (Badler[1979]), by smoothing its border in areas where insignificant axes are likely to be generated. Another approach developed by Duda and Hart (Duda[1973]) involves filtering the skeleton of the MAT by eliminating the branches that display a rapid variation in radii. The validity of this idea stems from the fact that insignificant branches of the MAT are generally made up, at one end, of small disks representing superfluous details and, at the other end, of large disks corresponding to major parts of the shape. Other methods, which use more sophisticated noise measuring criteria (Ho[1986, Cordella[1986]) to produce more accurate elimination, are less appealing since they require excessive computation time.

An efficient approach to this problem consists in defining a branch of the MAT as superfluous (i.e. caused by contour noise, or due to the digitizing process) whenever the ratio of its length over the length of the corresponding section of the shape's contour (called "boundary axis weight" in Blum[1978]) is above a specified threshold. This threshold should act as a resolution parameter in the sense that an increase in its value will give rise to a coarser approximation of the shape. This approach, although simpler than the one suggested in Ho[1986], which consists of geometrically measuring the degree of significance of contour details, brings about similar results in terms of smoothing efficiency. Figure 1 illustrates some examples of representation after application of the smoothing procedure. In order to allow the reader to foresee the effects of threshold variations, the value of the boundary axis weight is indicated for every section of the MAT.

### 2.3 Definition of deformable primitives.

As previously explained, trying to define new deformable primitives aims first at improving the local consistency between the shape under consideration and the primitive, and, second, at maximizing the spatial contribution of every primitive to the inside of the shape. In Blum[1973], the author introduces an axial representation in which three basic types of primitives are defined, corresponding respectively to convex, concave and straight-line symmetry (as illustrated in figure 2). A common feature of these primitives is their property of rectilinear symmetry, which makes them particularly appealing in the context of MAT representation.

Starting from these three basic forms, it thus seems possible to propose the following procedure for shape description:

- first replace the skeleton of the MAT by a piecewise linear approximation in order to allow further detection of rectilinear symmetries.
- then search each rectilinear segment for the presence of maximum size Blum primitives, on the basis of the radii values.

The following paragraph describes in detail the phases of the proposed procedure.

#### 2.3.1 Rectilinear approximation of the skeleton.

Numerous techniques exist for the computation of polygonal approximations of curves (Ballard[1982]). However, most of these techniques are suited to the case of simple curves only, and are inadequate in cases of multiple branch skeletons. Sklansky's algorithm (Sklansky[1980]) appears to be an effective solution for such cases, because of its sequential exploration of the skeleton. Starting from an initial vertex, the technique looks for the longest rectilinear segment maintaining the absolute approximation error below a specified threshold. Whenever the threshold is about to be exceeded, a vertex is placed and a new segment is initiated. Initial vertices are selected in order of decreasing radius, from among the yet unexplored positions. Figure 1 illustrates some examples of the effect of polygonal approximation (dots indicate positions of vertices found by the algorithm).

#### 2.3.2 Definition and modelling of primitives.

In order to detect the presence of complex primitives (such as Blum's) in the axial representation of shape, it is first necessary to establish a model of such primitives whose formulation remains consistent with this axial representation. As will be shown in this paragraph, Blum primitives may be considered as forms resulting from basic deformations of a disk. The deformations can be directly identified on the MAT of a shape, thus providing the means for a more concise description.

Given a two-dimensional shape, whose contour is represented parametrically by:

$$x = p_1(u)$$
$$y = p_2(u) \tag{2.1}$$

its deformation can be defined, in general, by two transformations:

$$X = f_1(x, y) = q_1(u)$$
$$Y = f_2(x, y) = q_2(u) \tag{2.2}$$

The resulting transformations affecting tangents is defined by:

$$\begin{pmatrix} \delta X/\delta u \\ \delta Y/\delta u \end{pmatrix} = \begin{pmatrix} \delta f_1/\delta x & \delta f_1/\delta y \\ \delta f_2/\delta x & \delta f_2/\delta y \end{pmatrix} \begin{pmatrix} \delta x/\delta u \\ \delta y/\delta u \end{pmatrix} \tag{2.3}$$

where the determinant of the Jacobian matrix represents the deformation affecting an infinitesimal surface patch.

Since the medial axis transform describes shapes by means of a succession of disks of varying radii, a deformation aimed at summarizing this description should duplicate the same process of combined dilation and translation of disks. One way to achieve this, in the case of Blum primitives, consists of defining a deformation process, which when applied to the initial disk $D_0$ of the MAT (see figure 3) produces the contour of the corresponding primitive. Restricting ourselves to the class of tangent preserving transformations, a reasonable choice for transformation (2.2) is:

$$X = a(y)x + b(y)$$
$$Y = a(y)y \tag{2.4}$$

Referring to figure 4, if $P$ and $Q$ designate a pair of corresponding points under transformations (2.4), the fact that tangents at $P$ and $Q$ have identical orientations implies that $a(y)$ represents the scaling, in the MAT representation, of the disk $D_Q$ corresponding to $Q$ with respect to $D_0$ and that $b(y)$ relates to the translation from $D_0$ to $D_Q$. Given these observations, transformations (2.4) are limited to the section $[y_i, y_f]$ of $D_0$, and shape continuity imposes:

$$a(y_0) = 1$$
$$b(y_0) = 0$$
$$a(y_f) = R_f/R_0$$
$$b(y_f) = b_f \tag{2.5}$$

The choice of this type of transformations is particularly appealing since its parameter $a(y)$ and $b(y)$ can be easily estimated owing to the natural similarity between expression (2.4) and the spatial distribution of disks along the MAT. Finally, the three types of primitives of figure 2 lead to the three basic types of transformations illustrated in figure 5. Convex symmetry primitives correspond to a strictly decreasing tangent slope with $y_f < y_i$, while concave symmetry primitives correspond to an increasing tangent slope with $y_f > y_i$. Straight line symmetry constitutes a limit situation between the two previous ones, with $y_f = y_i$. These transformations thus represent prototypes of evolution which, if properly detected in rectilinear segment of any MAT description, signal the presence of the corresponding primitives. Furthermore, if we restrict the prototypes of $a(y)$ and $b(y)$ to first order functions:

$$a(y) = a_0|y| + a_1$$
$$b(y) = b_0|y| + b_1 \tag{2.6}$$

each primitive can be completely specified by a very concise list of characteristics:

- the coordinates $(x_0, y_0)$ and the radius $R_0$ of the initial disk $D_0$
- the orientation $\theta_0$ of the axis of symmetry
- the extreme positions $y_i$ and $y_f$
- the two parameters $a_0$ and $b_0$

### 2.3.3 Detection of primitives in the MAT description.

Once the MAT description of a shape has been segmented into rectilinear sections, the evolution function $a(y)$ and $b(y)$ along each section must be evaluated in order to isolate maximum size primitives. This is easily done, since $a(y)$ represents the scaling of the disks of the MAT and $b(y)$ relates to their translation, with respect to an arbitrary initial position. Figure 6(a) illustrates an example of functions $a(y)$ and $b(y)$ evaluated along the main axis AB of the shape of figure 1(b). These functions must then be segmented into prototype sections of the kind illustrated in figure 5. In the special case of the first-order model (2.6), the partition is established through polygonal approximation (eventually preceeded by some filtering in order to reduce the influence of noise). Figure 6(b) illustrates the results obtained with the functions of figure 6(a), where each segment in the approximation of $a(y)$ and its corresponding segment in $b(y)$ represent an instance of either a convex, a concave or a straight-line primitive depending upon their respective slopes. Consecutive segments of $a(y)$ and $b(y)$ share a vertex of the MAT which corresponds to a disk common to the two associated primitives (hinge component). This disk roughly represents the amount of overlap between the two primitives.

One can notice that among all the rectilinear segments, some can be eliminated since they correspond to primitives with small spatial contributions. It is also important to note here that the significance of a given pair of segments on the $a(y)$ and $b(y)$ curves is not necessarily related to their lengths. Other factors are also taken into account such as the slopes of $a(y)$ and $b(y)$ and the size of the hinge components. For example, in figure 6(b), segments 1-2 to 4-5 are found to be the least significant. Consequently their elimination from the shape description creates an error which corresponds to the upper part of the light bulb, as shown in figure 7(b). Referring to the prototypes of figure 5, one can verify that segment 5-6 is a convex primitive while segment 6-7 is a concave primitive. Segment 9-10 is of a particular kind since it appears as a point in the $a(y)$ curve. This segment corresponds to the lower part of the bulb which can be represented by a straight-line primitive with constant radius circles (see figure 7(b)).

## 3. Results and discussion.

Figure 7 illustrates some final descriptions created by the proposed representation procedure. As can be observed, a very compact description is obtained for fairly complex shapes. These examples used a first-order model of deformations which explains the appearance of certain discontinuities in the approximations. Better results could be obtained when using second or higher order models.

The accuracy of the description is controlled by the specified tolerances applied during the rectilinear approximation of the skeleton and of $a(y)$ and $b(y)$. Of critical importance to the quality of the representation is the stability of the rectilinear approximation technique used in each case.

The performance of the representation under rotation depends on two factors. First, the initial axial representation should remain invariant under rotation since the determination of the deformed primitives essentially relies on it. It has been observed that the circular MAT computed with the Danielsson's algorithm (Danielsson[1980]) has a relatively stable behavior under any rigid transformation. Second, the MAT segmentation procedure should also be insensitive to rotation. The procedure actually used in this work (section 2.3.1) does not fully meet this requirement. This limitation results in some instability of component locations relative to rotation. However, this problem could be solved by using a better segmentation procedure.

Figure 7(a) shows a shape to which two notches have been added in order to simulate contour noise. As one can notice, these two notches have only local effect on the global representation. This can be viewed as a relative immunity to noise since the presence of these notches leads to the addition of two independent disks, which do not interfere with the rest of the description. This robustness to small variations of the contour is also apparent in figure 7(b). The presence of small contour details corresponding to the threads of the bulb did not affect the extraction of the more significant straight-line primitive 9-10.

No consideration of connectedness was taken into account in this work and this explains the unnatural results in cases of multiply connected shapes (figure 7c). However considerably simplified representations may be obtained by segregating such shapes into simply connected components.

At this point, one can say that the proposed representation satisfies the criteria of rich local support and subsumption in the following manner:

- first, the use of deformable primitives provides more flexibility which translates into a better local consistency with the shape under consideration.

- second, the choice of rectilinear-symmetry primitives allows the replacement of whole sections of the MAT by a single component, thus reducing the number and the mutual overlap of the descriptive components.

It is believed that this new type of representation, through the use of primitives which are more naturally related to the shape they represent, can provide more efficient descriptions than classical axial representations for such visual tasks as matching and recognition.

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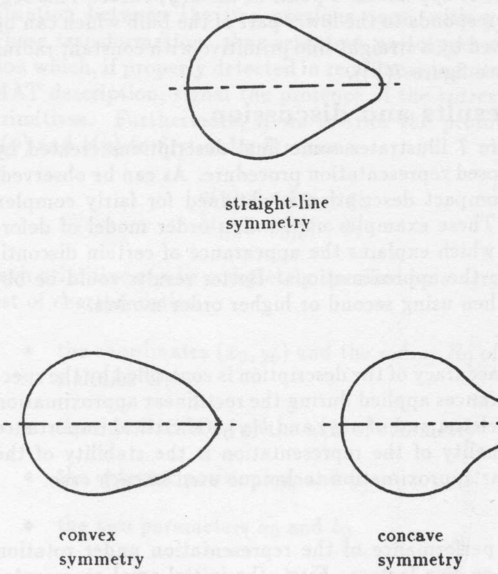


Figure 2. The three Blum's primitives.

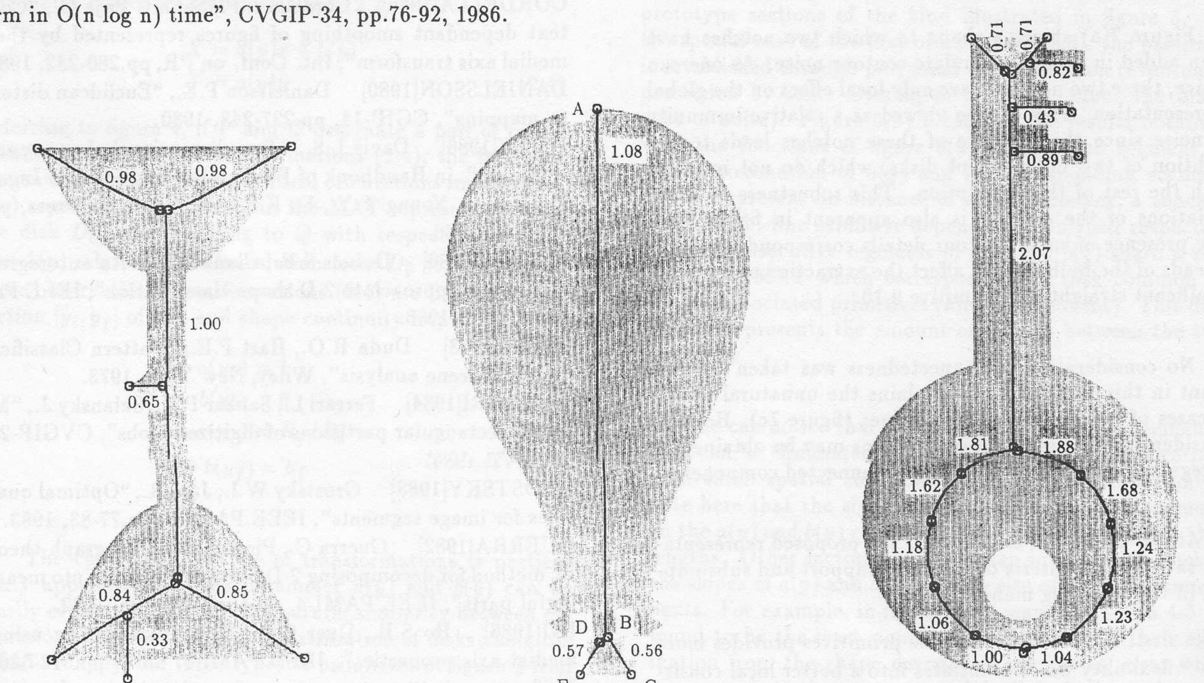


Figure 1. MAT representations after smoothing

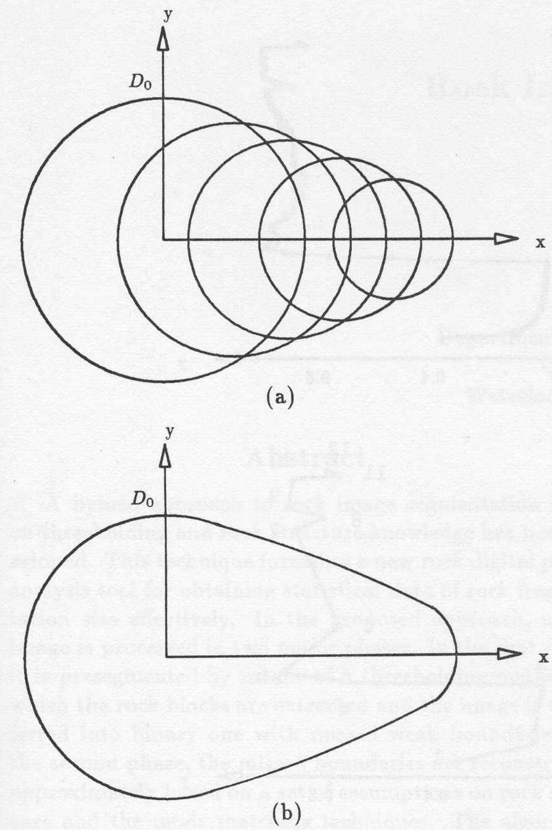


**Figure 3.** (a) Representation of part of a shape by means of successive covering. (b) representation by means of deformation.

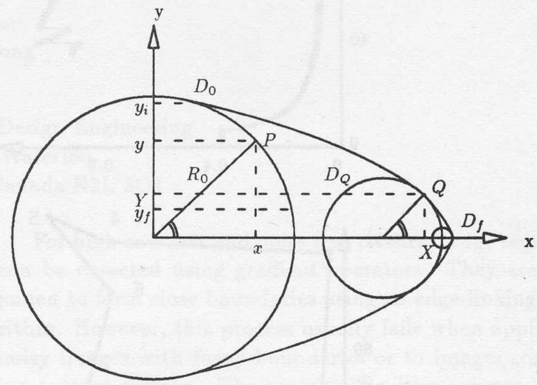


**Figure 4.** Characteristics of primitive deformation.

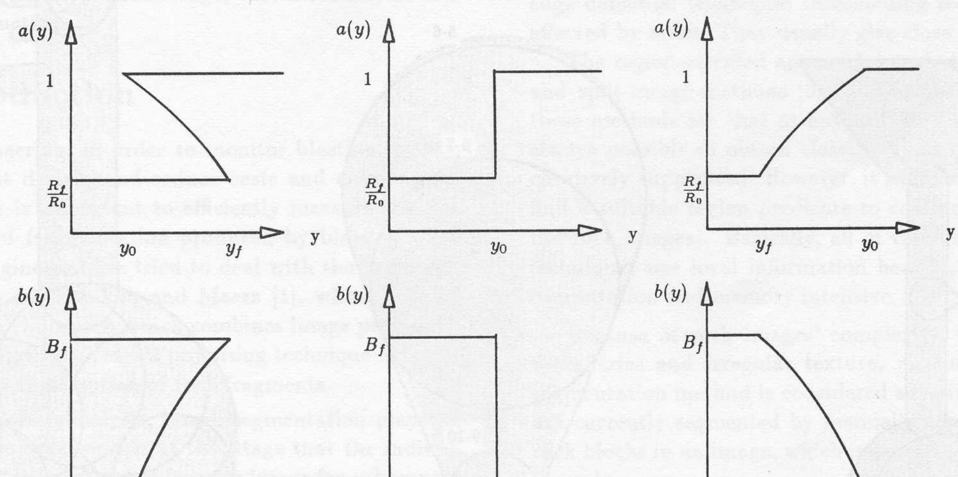


**Figure 5.** Prototypes of the three basic types of transformation.

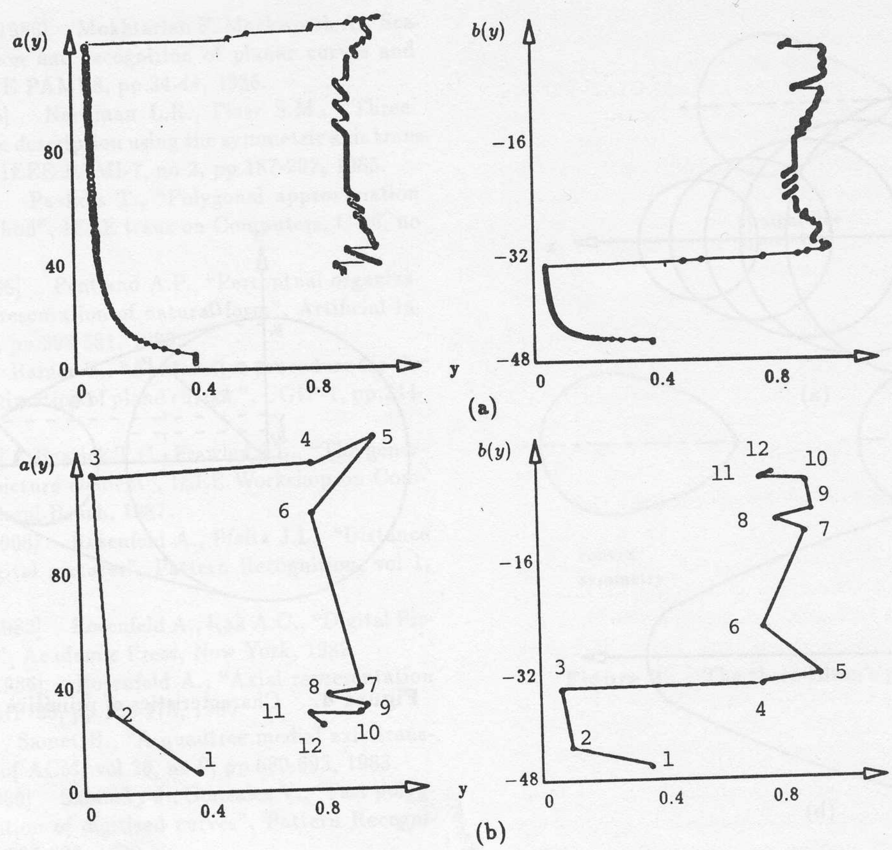


**Figure 6.** Detection of maximal primitives. (a) $a(y)$ and $b(y)$ evaluated on a real shape. (b) polygonal approximation of $a(y)$ and $b(y)$.

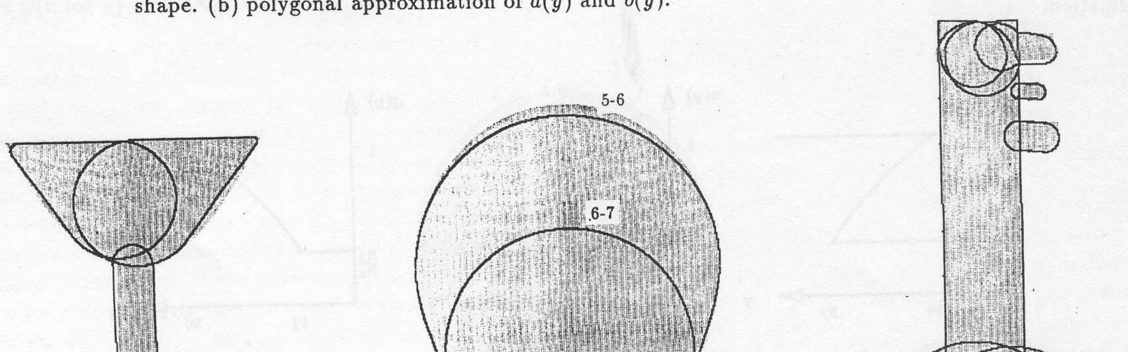


**Figure 7.** Representation of shapes with deformable primitives.