

An Adaptive Convex Projection Method for Resolution Enhancement of Tomographic Images

Shyh-shiaw Kuo * and Richard J. Mammone

Department of Electrical and Computer Engineering
Rutgers University, Piscataway, New Jersey 08855

Abstract

The Row Action Projection (RAP) method is used to increase the spatial resolution of images reconstructed via the Filtered Back Projection (FBP) algorithm. A new implementation of RAP is introduced which is computationally efficient and facilitates local adaptation of the projection operators. The local mean value as well as minimum and maximum bounds are used as constraints. The method is proposed to provide a *zoom-in* capability which yields a high resolution estimate of a specified region of the image. Computer simulations demonstrate the new method to be very effective in recovering high order spectral components of designated regions of the reconstructed image.

1 Introduction

Performance of the Filtered Back Projection (FBP) method of image reconstruction [1, Chap.3] approaches optimality in the mean squared sense for the parallel beam reconstruction algorithm [2]. It is desirable to maintain this level of performance when the effective processing bandwidth W_p is less than the image bandwidth W . This situation may arise when the reconstruction filter of the FBP method is deliberately truncated to control the signal-to-noise ratio (SNR) of the reconstructed images, or when the width of the detector is relatively large so that $W_p < W$.

The tradeoff between the rms error and spatial resolution in the reconstructed image is given by [3],

$$\epsilon \propto \left[\frac{W_p}{W} \right]^{\frac{1}{2}} \sigma, \quad (1)$$

where ϵ is the root mean squared error (rms) of the reconstructed image and σ is the standard deviation of the measurement noise. It is obvious from equation (1) that the rms error can be reduced by decreasing the processing bandwidth W_p . However, this will sacrifice the spatial resolution due to the elimination of the high frequency information. The reconstruction error is also dependent on the intensity of the beam as shown by Brooks and Dichiuro [2, eq.(10)]. Thus, another way to reduce the reconstruction error is to increase the intensity of the beam. Unfortunately, a decrease in reconstruction error by a factor of

two requires an increase in dose by a factor of eight, i.e., the relationship is third order [2]. Hence, this is generally prohibitive in practice. In addition to the physics underlying equation (1), the resolution of the reconstructed image is also limited by the process of windowing the sampled data. A smoothing window, such as a hamming window [4], is generally applied during the reconstruction process in order to reduce the ringing artifact in the reconstructed image. The use of a smoothing window results in an additional loss of resolution of the reconstructed image.

In this paper we present a method to increase the spatial resolution of the reconstructed image by constrained deconvolution or equivalently restoring the missing high frequency information. Image restoration problems of this type have been successfully solved via the adaptive Row Action Projection (RAP) algorithm [5] which is a subset of the method of Projection Onto Convex Sets (POCS) [6, Chap.2]. The new method is implemented by applying a sequence of local projection operators as opposed to the more traditional use of global projection operators.

At this point, we would like to clarify the ambiguity of the use of the word *projection*. In tomographic imaging, the word *projection* usually denotes a collection of line integrals in which the integration paths are straight lines passing through the object. In POCS, the word *projection* onto a set is used to indicate the mathematical operation which yields the point in the set which is closest to a given point. In order to avoid confusion of the term *projection*, we shall use the term *projection* as it applies to POCS and use the term *raysum* [7] to denote the physical projection of the tomographic imaging process.

The problem of resolution enhancement of the FBP reconstructed images is formulated in section 2. The approach of applying the RAP algorithm to the problem is presented in section 3. Numerical results are given in section 4, and finally, the summary and conclusions of the paper are given in section 5.

2 Problem Formulation

In this section, a brief review of the FBP method of image reconstruction is given in order to introduce the notation which will be used in the paper and to illustrate the details of the implementation. It is well known [1, 8] that the ideal back-projection image $f_b(x, y)$ is related to the original image $f(x, y)$ by the two dimensional symmetric convolution

*Current affiliation: AT&T Bell Labs, Murray Hill, New Jersey 07940.

kernel $\frac{1}{r}$, that is

$$f_b(x, y) = f(x, y) ** \frac{1}{r}, \quad (2)$$

where $r = \sqrt{x^2 + y^2}$ and $**$ denotes the two-dimensional convolution. The frequency domain representation of equation (2) in polar coordinates is

$$\rho \cdot F_b(\rho, \theta) = F(\rho, \theta), \quad (3)$$

where ρ is the two-dimensional Fourier transform of r .

Hence, the original image $f(x, y)$ can be reconstructed from the back-projection image by applying a 2-D filter whose Fourier domain representation in polar coordinates is given by

$$H(\rho) = \rho.$$

The two-dimensional transform can be implemented by calculating N one-dimensional transforms, where N is the number of raysums, as stated by the Fourier Slice Theorem [1]. The filtered raysum is denoted as

$$Q_\theta(R) = \int_{-W}^W S_\theta(\omega) |\omega| H_w(\omega) e^{j2\pi\omega R} d\omega, \quad (4)$$

where $S_\theta(\omega)$ is the Fourier transform of the raysum collected at angle θ and $H_w(\omega)$ is a 1-D rectangular window function which characterizes the processing bandwidth, W_p . If the processing bandwidth is equal to the bandwidth of the image, i.e. $W_p = W$, then $H_w(\omega)$ is unity. The FBP reconstructed image $f_{FBP}(x, y)$ is obtained by back projecting the sequences $Q_{\theta_i}(R)$ [1] as

$$f_{FBP}(x, y) = \frac{\pi}{K} \sum_{i=1}^K Q_{\theta_i}(x \cos \theta_i + y \sin \theta_i), \quad (5)$$

where the K angles θ_i are those for which the raysums are known. The discrete filtered raysums $Q_\theta(n\tau)$ are zero-padded [1, Chap.3] in order to avoid the *dishing* and *dc shift* artifacts.

Consider the situation where the effective processing bandwidth W_p is less than the bandwidth W of the true image. In this case, the 1-D window function in equation (4) is given by,

$$H_w(\omega) = \text{rect}\left(\frac{\omega}{W_p}\right) = \begin{cases} 1 & \text{for } |\omega| < W_p/2 \\ 0 & \text{for } |\omega| > W_p/2 \end{cases} \quad (6)$$

This corresponds to a modification of equation (3) by a 2-D window function given by,

$$\text{circ}\left(\frac{\rho}{2\rho_0}\right) = \begin{cases} 1 & \text{for } |\rho| < \rho_0 \\ 0 & \text{for } |\rho| > \rho_0 \end{cases} \quad (7)$$

If we denote the 2-D Fourier transform of the FBP reconstructed image as $G(\rho, \theta)$, then we have

$$G(\rho, \theta) = F(\rho, \theta) \cdot \text{circ}\left(\frac{\rho}{2\rho_0}\right) \cdot H_{sw}(\rho), \quad (8)$$

where $H_{sw}(\rho)$ is a two dimensional smoothing window. The space domain representation of equation (8) is

$$g(x, y) = f(x, y) ** h(x, y), \quad (9)$$

where $g(x, y)$ is the bandlimited reconstructed image via the FBP algorithm and $h(x, y)$ is the inverse Fourier transform of the multiplication of the two window functions in equation (8), i.e. $h(x, y) = \mathcal{F}^{-1}[\text{circ}(\frac{\rho}{2\rho_0}) \cdot H_{sw}(\rho)]$.

The problem of finding $f(x, y)$ given $g(x, y)$, as specified by equation (9), will be called a spatial resolution enhancement of the FBP reconstructed image. This problem is now one of image restoration which can be solved numerically by first transforming equation (9) into a discrete set of equations. Thus in matrix notation we have,

$$\mathbf{g} = \mathbf{H}\mathbf{f}, \quad (10)$$

where $\mathbf{g}$ and $\mathbf{f}$ are the lexicographic row-ordered vectors of the discretized versions of g and f respectively, and $\mathbf{H}$ is the degradation operator of the reconstruction system. The matrix $\mathbf{H}$ is generally rank-deficient or ill-conditioned due to the presence of spectral zeros in the continuous operator. In addition, the set of linear equations given by equation (10) will be inconsistent due to noise and modeling inaccuracies. Thus, the direct inverse is not applicable for solving the above set of linear equations. It is well known [6] that the ill-posed nature of the problem can cause an excessive error in $\hat{\mathbf{f}}$, the estimate of $\mathbf{f}$, due to even an extremely small error in the measured image $\mathbf{g}$. The problem of amplifying the measurement noise can be ameliorated by the use of constraints on $\hat{\mathbf{f}}$. The goal of enforcing these constraints is to reduce the possibility of large deviations in the estimate $\hat{\mathbf{f}}$ from what is known *a priori* about the actual image $\mathbf{f}$.

3 The New Method

POCS is an iterative technique to find a common point $\mathbf{f}$ which satisfies a set of constraints, each of which forms a convex set C_i in a Hilbert space $\mathfrak{R}$. The common point $\mathbf{f}$ can be found by alternatively projecting onto the convex sets C_i via the corresponding projection operator P_{C_i} . That is,

$$\mathbf{f}^{(k+1)} = P_{C_m} P_{C_{m-1}} \dots P_{C_1} \mathbf{f}^{(k)} = \mathbf{P}_C \mathbf{f}^{(k)}, \quad (11)$$

or more generally,

$$\mathbf{f}^{(k+1)} = T_{C_m} T_{C_{m-1}} \dots T_{C_1} \mathbf{f}^{(k)} = \mathbf{T}_C \mathbf{f}^{(k)}, \quad (12)$$

where $T_{C_i} = I + \lambda_i(P_{C_i} - I)$, $0 < \lambda_i < 2$, k denotes the iteration, and λ_i is the relaxation factor. Note that equation (11) corresponds to the case of unity relaxation (i.e. $\lambda_i = 1$) of the generalized expression (12). The initial guess $\mathbf{f}^{(0)}$ can be any point in $\mathfrak{R}$. It also should be mentioned that the P_{C_i} can be either linear or nonlinear projection operators.

3.1 The Row Action Projection Method

The RAP algorithm which was originally developed by

Kaczmarz [9] forms a solution to a set of linear equations $\mathbf{g} = \mathbf{H}\mathbf{f}$ by iterative orthogonal projections onto the hyperplanes specified by each equation. The method converges to the intersection of all the hyperplanes. The RAP update equation is given by,

$$\mathbf{f}^{(k+1)} = \mathbf{f}^{(k)} + \lambda_k \frac{g_{i_k} - \langle h_{i_k}, \mathbf{f}^{(k)} \rangle}{\|h_{i_k}\|^2} h_{i_k}, \quad (13)$$

where λ_k is the relaxation factor, g_i is the i^{th} element of the vector $\mathbf{g}_{M \times 1}$, h_i is the i^{th} row of the matrix $\mathbf{H}_{M \times M}$, $\langle \cdot, \cdot \rangle$ is the standard inner product between two vectors, $i_k = k(\text{mod } M)$ is the cyclic control parameter, $\|\cdot\|$ is the L_2 norm of a vector defined as $\|X\|^2 = \sum_{i=1}^N x_i^2$.

Kaczmarz [9] has shown that the iteration process given by equation (13) converges to the solution of equation (10) when $\mathbf{H}$ is a square nonsingular matrix, i.e., $\mathbf{H}$ has an inverse, and the relaxation parameter is unity. Proof of RAP's convergence for the consistent [10] and inconsistent [11] cases have also been demonstrated. The conventional Kaczmarz algorithm expressed by the equation (13) converges to the pseudoinverse or least square solution when the initial estimate $\mathbf{f}^{(0)}$ is in the range space of $\mathbf{H}^T$ [12] i.e.,

$$\lim_{k \rightarrow \infty} \mathbf{f}^{(k)} = \mathbf{H}^\dagger \mathbf{g}, \quad (14)$$

where $\mathbf{H}^\dagger$ denotes the pseudoinverse of $\mathbf{H}$. It has been shown that this iterative approach to obtain the pseudoinverse is computationally attractive with respect to alternate methods [13]. The method of RAP has been applied to the problem of image restoration [5] and adaptive filtering [14]. The Kaczmarz algorithm is also the basis for the Algebraic Reconstruction Technique (ART) of image reconstruction [8, Chap.11].

We extend the use of the RAP algorithm for resolution enhancement of FBP reconstructed images by recognizing that it is a form of the POCS method [7], since each hyperplane is a linear manifold which is closed and convex. Therefore the RAP algorithm can be generalized to include nonlinear constraints in each iteration, i.e., equation (13) can be rewritten as

$$\mathbf{f}^{(k+1)} = \mathbf{P}_C \mathbf{f}^{(k)} + \lambda_k \frac{g_{i_k} - \langle h_{i_k}, \mathbf{P}_C \mathbf{f}^{(k)} \rangle}{\|h_{i_k}\|^2} h_{i_k}, \quad (15)$$

where $\mathbf{P}_C = \mathbf{P}_{C_m} \mathbf{P}_{C_{m-1}} \dots \mathbf{P}_{C_1}$ is the concatenated projection operator which describes any *a priori* constraints.

We would like to distinguish RAP from ART which are very similar methods but are used for different purposes. ART, as the name implies, is a technique for image reconstruction and works directly on the raysums. RAP is a technique for image restoration and works on the mathematical derived image model as given by equation (9). The linear equations used in ART and RAP are set up according to the physical imaging system and the mathematical model, respectively. The conventional update equation in ART [7, eq.(31)] is based on the estimated raysums and the weights which depend on the length of the i^{th} ray through the pixel. The update equation generally has to be modified according to the physical imaging system [1, Chap.7].

However, the update equation (13) in RAP only depends on the distance from the previous estimate to the next equation. In addition, unlike ART, the image restoration problem solved by RAP is decomposable. This will be shown in the next section.

3.2 2-D Implementation of RAP

The RAP algorithm given by equation (13) can be more efficiently implemented by operating on the 2-D image and the 2-D Point Spread Function (PSF) directly as opposed to the usual column or row stacked approach. This can be seen in the following. Consider the case where the support of the PSF matrix is a 3×3 window as indicated by the solid outline in figure 1(a). In this case, every row of the matrix $\mathbf{H}$ in equation (10) contains only 9 nonzero elements. Each pixel $g(i, j)$ of the blurred image $\mathbf{g}$ corresponds to a specific equation of the set given by (10). The RAP algorithm, equation (13), requires only those 9 pixel values in $\mathbf{f}$ which correspond to the 9 nonzero elements in $\mathbf{h}_p$ which cover the support of the PSF. Hence, the 2-D formulation of the RAP algorithm can be written as:

$$\hat{f} = \begin{cases} \hat{f}(m, n) + \lambda \frac{\varepsilon(i, j)}{\|h\|^2} h(i - m, j - n) & \text{if } \hat{f}(m, n) \in S_h \\ \hat{f}(m, n) & \text{otherwise} \end{cases}, \quad (16)$$

where S_h is the support of $\mathbf{h}$ and

$$\varepsilon(i, j) = g(i, j) - \sum_{m, n \in S_h} h(i - m, j - n) \hat{f}(m, n), \quad (17)$$

$$\|h\|^2 = \sum_{m, n \in S_h} h(m, n)^2. \quad (18)$$

For example, S_h is a 3×3 window for the case shown in figure 1. The equation selected next for the projection operation can be obtained by shifting the PSF matrix one pixel (down, for example) as indicated in figure 1(b). This shift corresponds to a specific ordering (cycling strategy) of the projections onto the equations given (10). Hence, the RAP algorithm can be implemented in a manner similar to a 2-D convolution and facilitates the local projection operator which provides three major advantages. First, the RAP method can be alternately exploited by implementing many non-overlapping projections in parallel [5]. Second, this algorithm can be applied adaptively to any specific local regions. Third, it facilitates the use of adaptive constraints and different cycling strategies, i.e. the selection of the next block to update. We shall discuss these features of RAP in the following sections.

3.3 Smoothness Constraint and Cycling Strategy

Since RAP makes use of the local projection operators, the update strategy can be adaptive according to local 2-D features of the image. Sezan and Tekalp [15] have shown that scene-dependent adaptive processes can be facilitated by segmenting the image into regions of comparable gray level according to the local variance. Regions which are basically uniform-, texture- and edge-like are denoted by

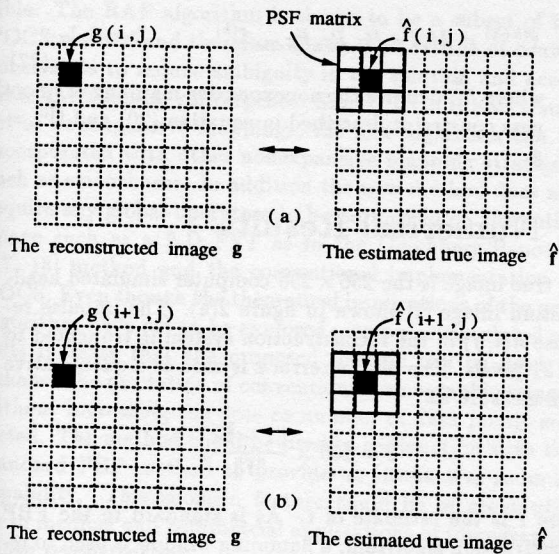


Figure 1: 2-D implementation of RAP algorithm: (a) Project onto one of the equations in the set $g = Hf$, (b) Project onto next equation in the set of equations $g = Hf$.

R_1 , R_2 and R_3 respectively and are defined as:

$$\begin{aligned} R_1 &\equiv \{(i, j) \mid 0 \leq \sigma_g(i, j) \leq T_a\} \\ R_2 &\equiv \{(i, j) \mid T_a \leq \sigma_g(i, j) \leq T_b\} \\ R_3 &\equiv \{(i, j) \mid T_b \leq \sigma_g(i, j)\} \end{aligned} \quad (19)$$

where $\sigma_g(i, j)$ denotes the local variance of the blurred image $g(i, j)$. This can be computed over some rectangular window of pixels centered at (i, j) . The thresholds T_a and T_b are determined experimentally [15].

The smoothness constraint can be applied via the projection operator which restricts the local variance bound as given by Sezan and Tekalp [15]. Alternately, we found that neighborhood averaging provides a means of constraining regions to be smooth. This approach is simpler in the sense that it requires less arithmetic operations. The neighborhood averaging is given by

$$f(i, j) = \begin{cases} \bar{f}(i, j) & \text{if } f(i, j) \in R_1 \\ f(i, j) & \text{otherwise} \end{cases}, \quad (20)$$

where

$$\bar{f}(i, j) = \frac{1}{N} \sum_{(m, n) \in S} f(m, n), \quad (21)$$

S is some rectangular window of pixels centered at (i, j) , and N is the total number of pixels of $f(m, n)$ in that window. The neighborhood averaging has been shown to be a nonexpansive mapping and can be used in cooperation with POCS [16].

There is one equation in the set given by $g = Hf$ for each pixel in the blurred image g . The selection of which equation to project onto next is called the cycling strategy. One example of a cycling strategy is when projections are

made only along the edge-like region of an image via RAP. Thus the edges can be deblurred as much as possible. This can be expressed as a projection operator P_{C_e} of POCS as given by

$$P_{C_e} f = \begin{cases} f + \lambda \frac{g(i, j) - d^T f}{\|d\|^2} d & \forall g(i, j) \in R_3 \\ f & \text{otherwise} \end{cases}, \quad (22)$$

where d denotes the normal vector of the hyperplane which associates with the $(i, j)^{th}$ pixel of the blurred image g .

3.4 Implementation

The RAP method is applied to the problem of spatial resolution enhancement of the FBP reconstructed images as follows:

1. **System Identification:** The impulse response $h(x, y)$ of the reconstruction system can be calculated by using the inverse Fourier transform of the transfer function between the FBP reconstructed image and a known test image,

$$h(x, y) = IFFT \left[\frac{F_{FBP}(u, v)}{F_{test}(u, v)} \right], \quad (23)$$

where $F_{FBP}(u, v)$ and $F_{test}(u, v)$ represent the 2-D Fourier transform of the FBP reconstructed image and the test image, respectively. The magnitude of the sidelobes of $h(x, y)$ is generally much lower than that of the mainlobe. Keeping them increases the size of the PSF, which correspondingly increases the number of required computations at each iteration of RAP. Thus the truncated version of $h(x, y)$ is used during the restoration process in order to reduce the computational complexity.

2. **Working Region Selection:** By taking advantage of the local projection operator of the RAP algorithm, we can specify a region of the FBP reconstructed image for increasing resolution. This zoom-in feature not only reduces the computational complexity by reducing the number of pixels to be processed but also provides a practical utility because the resolution enhancement is generally desired for some specific region in the reconstructed image. The process of applying the RAP method to the designated region can be expressed as a cycling strategy. The projection operator $P_{C_{zoom}}$ is defined by,

$$P_{C_{zoom}} f = \begin{cases} f + \frac{g(i, j) - \langle h, f \rangle}{\|h\|^2} h & \text{if } g(i, j) \in Z_R \\ f & \text{otherwise} \end{cases}, \quad (24)$$

where h denotes the normal vector of the hyperplane which represents the $(i, j)^{th}$ pixel of g and Z_R is the designated zoom-in region.

3. **The Local Minimum and Maximum Bounds Estimation:** The minimum and maximum bound constraints can be expressed as a projection operator of POCS as

$$P_{C_b} f = \begin{cases} f_{max} & \text{if } f(i, j) > f_{max} \\ f_{min} & \text{if } f(i, j) < f_{min} \\ f(i, j) & \text{otherwise} \end{cases} \quad (25)$$

Since the dynamic range of a designated region of the image is generally less than that of the entire image, the local minimum and maximum bounds have to be estimated for the designated region. Based on the fact that the rate of convergence is improved as the accuracy of the estimated bounds increases, the minimum and maximum bounds for Z_R can be estimated by the following steps:

Step 1: Z_R is segmented to separate the high intensity region from the background grey levels.

Step 2: Set $k = 1$.

Step 3: The RAP method is applied to Z_R for k iterations. The smoothness and the global minimum & maximum bound constraints are imposed. The initial estimate $f^{(0)}$ is set to the reconstructed image g .

Step 4: According to the segmentation in step 1, the average value of the background in the estimated image obtained from the previous step is computed and it is used as the tentative minimum bound $\hat{f}_{min}$. The largest value of region Z_R in the estimated image is found and it is used as the tentative maximum bound $\hat{f}_{max}$ if it is smaller than the global maximum bound.

Step 5: The RAP method is again applied to Z_R with tentative minimum and maximum bounds, $\hat{f}_{min}$ and $\hat{f}_{max}$, imposed. The initial estimate $f^{(0)}$ is again set to g . RAP is run for M iterations, where M is a small fixed number. The sum of the seminorm is calculated,

$$L_k = \sum_{m=1}^M S_m, \quad (26)$$

where S_m denotes the seminorm after the m^{th} iteration. The seminorm is defined as the sum of the residuals

$$S_m = \sum_{i,j \in Z_R} |g(i, j) - \langle h, f^{(m)} \rangle|,$$

where $f^{(m)}$ denotes the estimated image after the m^{th} iteration.

Step 6: If $L_k - L_{k-1} > 0.1$ then STOP, i.e. the local minimum & maximum bounds estimation stage is terminated when the sum of the seminorm increases. Otherwise, set $k = k + 1$ and go to Step 3.

The resulting tentative minimum and maximum bounds will be used as the f_{max} and f_{min} in equation (25) for the designated region Z_R .

4. Image Restoration: The true image can be restored by the following iterative process:

$$f^{(k+1)} = M_s P_{C_b} P_{C_e} P_{C_b} P_{C_{zoom}} f^{(k)}, \quad k = 0, 1, \dots, \quad (27)$$

where M_s denotes the nonexpansive mapping of smoothness constraint described in equation (20) and $f^{(0)} = g$.

4 Numerical Results

The true image is the 256×256 computer simulated head-phantom image [1] shown in figure 2(a). The impulse response $h(i, j)$ of the reconstruction system is truncated to 31×31 pixels. The relative error ϵ is used as a quantitative error measure as

$$\epsilon = 100 \times \frac{\|f - \hat{f}\|}{\|f\|},$$

where $\hat{f}$ is the estimate of f . As is standard in the FBP reconstruction algorithm, a hamming window is used to reduce the ringing artifact in the reconstructed image. The experiment is designed to demonstrate the new algorithm's ability to reduce the reconstruction noise and increase the resolution of the reconstructed image at the same time. Figure 2(b) is the reconstructed image using the standard implementation of FBP algorithm with white Gaussian noise added to each raysum such that the SNR = 30 dB. It is obvious from figure 2(b) that the reconstructed image is quite noisy. However, the rms error of the reconstructed image can be reduced by decreasing the processing bandwidth W_p [3]. Figure 2(c) is the bandlimited FBP reconstructed image from the same noisy raysum data used in figure 2(b). Each raysum is bandlimited to $W_p = 0.4 \frac{1}{\tau}$, where τ is the sampling interval in each raysum. It is clear that the noise in the reconstructed image has been reduced but the resolution of the image is also reduced. For example, the two spots in the bottom are not discernible in figure 2(c). But the two spots can be restored successfully by using the new algorithm. The edge windowed by a box is also enhanced. The resulting image is shown in figure 2(d). Thus the new algorithm combined with the bandlimited FBP algorithm can reduce the rms error and simultaneously increase the resolution of the reconstructed image. The relative errors of the zoom-in regions of figures 2(b), (c) and (d) are summarized in table 1.

5 Summary and Conclusion

A new projection method is presented for the purpose of enhancing the resolution of reconstructed tomographic images. The proposed problem is transformed into one of image restoration problem and solved by regularized deconvolution. The new algorithm does not require the storage of the entire lexicographically stacked degradation matrix H because of the 2-D implementation of the RAP algorithm. This reduces not only the memory space requirements but also the computational complexity. Hence, the capability of processing large images adaptively or in parallel is pos-

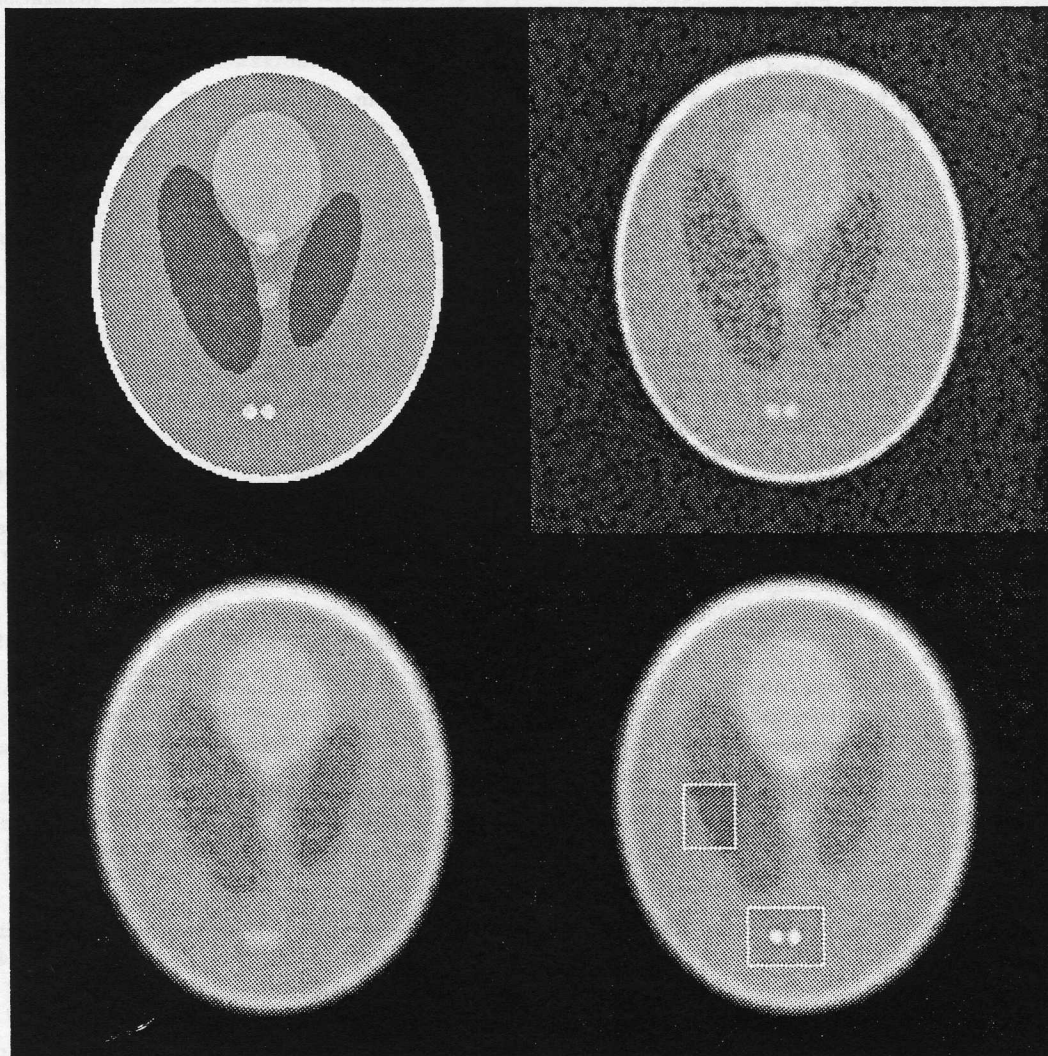
sible. The RAP algorithm is shown to be a subset of the POCS method and therefore allows the addition of convex constraints to reduce ambiguity in the solution and accelerate the speed of convergence. Since POCS is also a subset of nonexpansive mappings, the new algorithm can be incorporated with other nonexpansive mapping strategies such as smoothness. In addition the new method does not require any global operators to be performed on the entire image such as a 2-D FFT as in the Gerchberg-Papoulis [17, 18] method and the conventional implementation of POCS. Even though the theoretical implications of the new algorithm have yet to be explored, computer simulated results indicate that the proposed method can be used to enhance the resolution of conventional tomographic images without increasing the dose or number of data points collected. The method could be used in conjunction with the standard FBP method to provide an interactive zoom-in capability. This zoom-in feature could be of great utility in medical and commercial applications of tomographic image reconstruction.

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Table 1: The relative errors of the designated regions.

images	Standard FBP ($W_p = \frac{1}{\tau}$)	Bandlimited FBP ($W_p = 0.4\frac{1}{\tau}$)	New algorithm
edge	5.4 %	4.1 %	3.2 %
two spots	7.9 %	11.4 %	6.0 %



a	b
c	d

Figure 2: (a) The 256×256 head-phantom target image. (b) The standard FBP reconstructed image. (c) The bandlimited FBP reconstructed image. (d) The resolution of the image inside the boxes has been enhanced by using the new algorithm.