

# RP Transformation for Pattern Recognition \*

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## Abstract

To handle compound pattern recognition problem which deals with recognizing unconnected patterns and patterns with isolated noise, this paper presents a new approach, regional projection transformation (RPT). The compound pattern can be transformed into an integral object by the RPT. Two transformations have been described and analyzed in this paper: (1) diagonal-diagonal regional projection transformation (DDRPT), and (2) horizontal-vertical regional projection transformation (HVRPT). The patterns transformed from these two methods possess several important characteristics which facilitate the recognition of compound patterns. Parallel algorithms for both DDRPT and HVRPT have been developed in this paper. They can speed up computation and the recognition process.

## 1 Introduction

For well over a decade, parallel computing has been one of the most active and productive research fields in computer science [4]. Its importance in design of new computer architecture, as well as application to a wide range of areas including pattern recognition [1, 2, 6, 9], is beyond dispute. In this paper, a parallel algorithm has been developed to solve character recognition problem based on regional projection transformation (RPT).

Although contour analysis has been amenable to character recognition [5, 6], it still has a great difficulty in dealing with compound patterns including unconnected patterns and patterns with isolated noise. One of the remedial approaches is that the compound pattern will be transformed into an integral object. A

theoretical analysis about the compound and integral patterns will be stated in this paper.

Two transformations will be described and analyzed in this paper: (1) diagonal-diagonal regional projection transformation (DDRPT), and (2) horizontal-vertical regional projection transformation (HVRPT). The patterns transformed from these two methods possess several important characteristics which facilitate the recognition of compound patterns. Parallel algorithms for both DDRPT and HVRPT have been developed in this paper. They can speed up computation and the recognition process.

## 2 Integral Pattern and Compound Pattern

A digitized pattern  $\Omega$  is a finite set of pixels,  $\Omega = \{ \alpha_1, \alpha_2, \dots, \alpha_n \}$ , which can be viewed as a graph by graph theory.

### 2.1 Weighted Edge Graph (WEG)

**Definition 1** Let  $\Omega = \{ \alpha_1, \alpha_2, \dots, \alpha_n \}$  be a pattern. The structure of a pattern can be represented by a finite undirected graph termed by *weighted edge graph (WEG)*:

$$\begin{aligned} WEG &= \{V, E, D(E)\} \\ V &= \{v_1, v_2, \dots, v_n\} \\ E &= \{e_1, e_2, \dots, e_m\} \\ f(e_i) &= \{v_j, v_{j+1}\} \\ D(e_i) &= \overline{v_j, v_{j+1}} \\ \forall i &= (v_i \in \Omega). \end{aligned} \quad (1)$$

Eq. (1) implies that each pixel  $\alpha_i$  of the pattern  $\Omega$  is a node  $v_i$  of the WAG, and edges  $e_i$ 's are connected among all pairs of adjacent nodes. Each edge  $e_i = \{v_i, v_{j+1}\}$  is weighted with a distance between its endpoints  $D(e_i) = \overline{v_j, v_{j+1}}$ .

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## 2.2 Pattern Graph (PG)

**Definition 2** A WEG is called a *pattern graph (PG)* if all edges with higher weight  $\delta$  in the WEG have been removed such that

$$PG = \{V, E, D(E)\}$$

$$D(E) = \{D(e_i) \mid D(e_i) < \delta\} \quad (2)$$

where,  $\delta$  is a threshold depending on the structure of the specific pattern.

## 2.3 Integral Pattern and Compound Pattern

**Definition 3** A pattern is called *integral pattern* if its PG is a connected graph. Otherwise it is termed as *compound pattern*.

In fact, compound patterns can be categorized as two types: (1) unconnected pattern which is composed of several parts, (2) noise-patch pattern where several isolated patches of noise are embedded in it. By this definition, Patterns in Fig. 1(a) are compound patterns, while patterns in Fig. 1(b) are integral ones. Recognizing such compound patterns is much more difficult than processing the integral ones. To avert this problem, a compound pattern will be transformed into an integral object by the proposed method.

## 3 Regional Projection Transformation (RPT)

Two transformations have been developed: (1) diagonal-diagonal regional projection transformation (DDRPT), and (2) horizontal-vertical regional projection transformation (HVRPT).

The patterns transformed from these two RPT's possess several important characteristics which facilitate the recognition process. Although the shape of the pattern has been changed, it contains enough information to identify different objects. This can be evaluated in Section 5.

### 3.1 HVRPT and DDRPT

Let  $f(x, y)$  be a pattern with size  $N \times N$ , and  $R$  stand for an area of the pattern. Assume that  $f(x, y) = 0$  lies outside the pattern.  $\delta[\dots]$  denotes a delta function.  $t = x \sin \phi - y \cos \phi$  gives the Euclidean distance of a line from the origin. If the projection angle from the x-axis is  $\phi$ , the projection can be defined as follows [7]:

$$p(\phi, t) = \sum_R f(x, y) \delta[x \sin \phi - y \cos \phi - t]. \quad (3)$$

#### 3.1.1 HVRPT

The area of a pattern is divided into 8 sub-regions, symbolized by  $P, Q, R, S, T, U, V,$  and  $W$ . All pixels in the pattern are concentrated onto horizontal

and vertical lines, i.e.  $y = N/2$  and  $x = N/2$ , as shown in Fig. 2(a).

$$T_{HV} = P(315^\circ, t) \cup Q(315^\circ, t) \cup R(45^\circ, t) \cup S(45^\circ, t) \cup T(135^\circ, t) \cup U(135^\circ, t) \cup V(225^\circ, t) \cup W(225^\circ, t), \quad (4)$$

where,  $T_{HV}$  stands for the transformation of the whole pattern,  $P(315^\circ, t) - W(225^\circ, t)$  denote sub-transformations for sub-regions  $P - W$  respectively. 8 sub-projections can be calculated below:

$$P(315^\circ, t) = \sum_{x=0}^{N/2} f(x, \sqrt{2}t - x + \frac{N}{2})$$

for  $t = 0, 1, 2, \dots, \frac{N}{2}$

$$Q(315^\circ, t) = \sum_{x=0}^{N/2} f(x, \frac{N}{2} - \sqrt{2}t - x)$$

for  $t = 0, 1, 2, \dots, \frac{N}{2}$

$$R(45^\circ, t) = \sum_{x=0}^{N/2} f(x, x + \sqrt{2}t)$$

for  $t = 0, 1, 2, \dots, \frac{N}{2}$

$$S(45^\circ, t) = \sum_{x=0}^{N/2} f(x, x - \sqrt{2}t)$$

for  $t = 0, 1, 2, \dots, \frac{N}{2}$

$$T(135^\circ, t) = \sum_{x=N/2}^N f(x, \frac{N}{2} - \sqrt{2}t - x)$$

for  $t = 0, 1, 2, \dots, \frac{N}{2}$  (5)

$$U(135^\circ, t) = \sum_{x=N/2}^N f(x, \sqrt{2}t - x + \frac{N}{2})$$

for  $t = 0, 1, 2, \dots, \frac{N}{2}$

$$V(225^\circ, t) = \sum_{x=N/2}^N f(x, x - \sqrt{2}t + \frac{N}{2})$$

for  $t = 0, 1, 2, \dots, \frac{N}{2}$

$$W(225^\circ, t) = \sum_{x=N/2}^N f(x, x + \sqrt{2}t + \frac{N}{2})$$

for  $t = 0, 1, 2, \dots, \frac{N}{2}$

**Definition 4** Let  $\mathfrak{S}_{HV}$  be a pattern transformed from HVRPT defined by Eqs. (4) and (5), and  $\mathfrak{R}$  be a set of projection bases  $\mathfrak{R} = \{R_1, R_2, \dots, R_n\}$ . An HVRP pattern  $P_{HV}$  can be represented by

$$P_{HV} = \mathfrak{S}_{HV} \cup \mathfrak{R}. \quad (6)$$

Here, projection bases are two segments of horizontal ( $y = N/2$ ) and two segments of vertical ( $x = N/2$ ),  $n = 4$ . An example of HVRP pattern can be exemplified by Fig. 3(a).

### 3.1.2 DDRPT

Similarly, the area of a pattern is also divided into 8 sub-regions, but the bases onto which the pixels to be projected differ from that of the HVRPT. In this scheme, the projection bases are two diagonals (i.e.  $45^\circ$  and  $135^\circ$ ), all pixels in the pattern are concentrated onto these two diagonals as shown in Fig. 2(b). The eight sub-regions are symbolized by  $A, B, C, D, E, F, G$ , and  $H$  such that

$$T_{DD} = A(90^\circ, x) \cup B(90^\circ, x) \cup C(90^\circ, x) \cup D(90^\circ, x) \cup E(0^\circ, y) \cup F(0^\circ, y) \cup G(0^\circ, y) \cup H(0^\circ, y), \quad (7)$$

where,  $T_{DD}$  stands for the transformation of the whole pattern,  $A(90^\circ, x) - H(0^\circ, y)$  denote sub-transformations for sub-regions  $A - H$  respectively.

From Eq. (3), 8 sub-projections can be calculated as follows:

$$A(90^\circ, x) = \sum_{y=N-x}^N f(x, y)$$

$$\text{for } x = 0, 1, 2, \dots, \frac{N}{2}$$

$$H(90^\circ, x) = \sum_{y=x}^N f(x, y)$$

$$\text{for } x = \frac{N}{2}, \frac{N}{2} + 1, \dots, N$$

$$D(90^\circ, x) = \sum_{y=0}^x f(x, y)$$

$$\text{for } x = 0, 1, 2, \dots, \frac{N}{2}$$

$$E(90^\circ, x) = \sum_{y=0}^{N-x} f(x, y)$$

$$\text{for } x = \frac{N}{2}, \frac{N}{2} + 1, \dots, N$$

$$B(0^\circ, y) = \sum_{x=0}^{N-y} f(x, y)$$

$$\text{for } y = \frac{N}{2}, \frac{N}{2} + 1, \dots, N \quad (8)$$

$$C(0^\circ, y) = \sum_{x=0}^y f(x, y)$$

$$\text{for } y = 0, 1, 2, \dots, \frac{N}{2}$$

$$F(0^\circ, y) = \sum_{x=N-y}^N f(x, y)$$

$$\text{for } y = 0, 1, 2, \dots, \frac{N}{2}$$

$$G(0^\circ, y) = \sum_{x=y}^N f(x, y)$$

$$\text{for } y = \frac{N}{2}, \frac{N}{2} + 1, \dots, N.$$

**Definition 5** Let  $\mathfrak{S}_{DD}$  be a pattern transformed from DDRPT defined by Eqs. (7) and (8), and  $\mathfrak{R}$  be a set of projection bases  $\mathfrak{R} = \{R_1, R_2, \dots, R_n\}$ . A DDRP pattern  $P_{DD}$  can be represented by

$$P_{DD} = \mathfrak{S}_{DD} \cup \mathfrak{R}. \quad (9)$$

Here, projection bases are four segments of the  $45^\circ$  diagonal and  $135^\circ$  diagonal ( $n = 4$ ).

An example of the DDRP pattern can be exemplified by Fig. 3(b).

### 3.1.3 Characteristic of HVRP and DDRP patterns

A couple of very important characteristics of the patterns generated by the HVRPT and DDRPT can be presented in the following theorems, which play a major role in pattern recognition using contour information.

**Theorem 1** Either DDRP pattern or HVRP pattern does not contain any hole in it.

**Proof:** First, we prove the case of the DDC pattern. A DDC pattern  $P_{DDC}$  consists of 8 sub-patterns  $P_A - P_H$ , such that  $P_{DDC} = P_A \cup P_B \cup P_C \cup P_D \cup P_E \cup P_F \cup P_G \cup P_H$ . Furthermore, these eight sub-patterns are generated by eight sub-projections given in Eq. (8). Each sub-projection is made along either horizontal or vertical line. For instance, the sub-pattern  $P_A$  is made by  $A(90^\circ, x) = \sum_{y=N-x}^N f(x, y)$ , for  $x = 0, 1, 2, \dots, \frac{N}{2}$ . It reveals that all pixels in area  $A$  are projected onto a diagonal along the horizontal line which results no interval on the horizontal direction. Thus no hole exists in sub-pattern  $P_A$ . Similarly, Sub-patterns  $P_H, P_D$  and  $P_E$  do not contain any hole in them, since sub-projections in areas  $H, D$  and  $E$  result no intervals on the horizontal direction. Meanwhile, no intervals along the vertical direction occur in the areas  $B, C, F$  and  $G$  after projecting all pixels along the vertical.

Subsequently, the whole *DDC* pattern does not contain any hole because the sub-patterns do not contain any holes.

The case of the *HVC* pattern resembles the case of *DDC*. ■

**Theorem 2** Either *DDRP* pattern or *HVRP* pattern is an integral pattern.

The proof of this theorem is omitted.

Although the contour analysis has been amenable to character recognition, it still has a great difficulty dealing with *multiple-contour* problem which occurs in two cases [9]:

- (1) Compound patterns including unconnected patterns and patterns with isolated noise;
- (2) Patterns with internal contours.

The first case can be solved by Theorem 2. According to it the compound pattern is concentrated on an integral object to which the contour approach becomes applicable.

The second case can be handled according to the characteristic stated in Theorem 1. The pattern transformed from the proposed RPT's contains only one outer contour. It makes sense to simplify the process.

## 4 Parallel Algorithms

From the term "regional projection" of this approach, it is evident that both *HVRPT* and *DDRPT* have clear parallelism. All sub-regions can be processed at the same time. In this section, parallel algorithms for these RPT's have been developed.

### 4.1 Parallel Algorithm for *HVRPT*

Eight procedures for sub-regions *P* - *W* can be developed by Eq. (5). They can be executed concurrently. The parallel algorithm for the whole pattern can be presented in the following form.

#### Algorithm 1

```

parfor all pixels in a  $N \times N$  pattern do
  parbegin
    parfor  $x = 0$  to  $\frac{N}{2}$  do
      parfor  $y = \frac{N}{2}$  to  $N$  do
        Pixels in the sub-region P are projected
        onto the segment  $\alpha\beta_1$ 
        along the direction of  $315^\circ$  (Fig. 2(a));
        Pixels in the sub-region Q are projected
        onto the segment  $\alpha\beta_2$ 
        along the direction of  $315^\circ$ ;
      parfor  $y = 0$  to  $\frac{N}{2}$  do
        Pixels in the sub-region R are projected
        onto the segment  $\alpha\beta_2$ 
        along the direction of  $45^\circ$ ;
        Pixels in the sub-region S are projected
        onto the segment  $\alpha\beta_3$ 
        along the direction of  $45^\circ$ ;
    parfor  $x = \frac{N}{2}$  to  $N$  do
      parfor  $y = 0$  to  $\frac{N}{2}$  do

```

```

        Pixels in the sub-region T are projected
        onto the segment  $\alpha\beta_3$ 

```

```

        along the direction of  $135^\circ$ ;

```

```

        Pixels in the sub-region U are projected
        onto the segment  $\alpha\beta_4$ 

```

```

        along the direction of  $135^\circ$ ;

```

```

  parfor  $y = \frac{N}{2}$  to  $N$  do

```

```

    Pixels in the sub-region V are projected
    onto the segment  $\alpha\beta_4$ 

```

```

    along the direction of  $225^\circ$ ;

```

```

    Pixels in the sub-region W are projected
    onto the segment  $\alpha\beta_1$ 

```

```

    along the direction of  $225^\circ$ 
  parend

```

```

  parend

```

### 4.2 Parallel Algorithm for *DDRPT*

**Algorithm 2** The parallel algorithm for the *DDRPT* can be presented in the following simple form:

```

parfor all pixels in a  $N \times N$  pattern do
  parbegin

```

```

    Procedure A;

```

```

    Procedure B;

```

```

    Procedure C;

```

```

    Procedure D;

```

```

    Procedure E;

```

```

    Procedure F;

```

```

    Procedure G;

```

```

    Procedure H;
  parend

```

```

  parend

```

Procedures A - H correspond to eight sub-regions. They are written as follows:

#### Procedure A

```

parforallpixels in the sub-area A do
  parbegin

```

```

    parfor  $x = 0$  to  $\frac{N}{2}$  do

```

```

      for  $y = N - x$  to  $N$  do

```

```

        Pixels in the sub-area A are projected

```

```

        onto the segment  $\alpha\beta_1$  of the diagonal

```

```

        with  $135^\circ$  along the direction of  $270^\circ$ ;

```

```

      parend
    parend

```

#### Procedure B

```

parforallpixels in the sub-area B do
  parbegin

```

```

    parbegin

```

```

      parfor  $y = \frac{N}{2}$  to  $N$  do

```

```

        for  $x = 0$  to  $N - y$  do

```

```

          Pixels in the sub-area B are projected

```

```

          onto the segment  $\alpha\beta_1$  of the diagonal

```

```

          with  $135^\circ$  along the direction of  $0^\circ$ ;

```

```

        parend
      parend
    parend

```

#### Procedure C

```

parforallpixels in the sub-area C do
  parbegin

```

```

    parbegin

```

```

      parfor  $y = 0$  to  $\frac{N}{2}$  do

```

```

        for  $x = 0$  to  $y$  do

```

```

          Pixels in the sub-area C are projected

```

```

          onto the segment  $\alpha\beta_2$  of the diagonal

```

```

          with  $45^\circ$  along the direction of  $0^\circ$ ;

```

```

        parend
      parend
    parend

```

#### Procedure D

```

parforallpixels in the sub-area D do
  parbegin
    parfor  $x = 0$  to  $\frac{N}{2}$  do
      for  $y = 0$  to  $x$  do
        Pixele in the sub-area  $D$  are projected
        onto the segment  $\alpha\beta_2$  of the diagonal
        with  $45^\circ$  along the direction of  $90^\circ$  ;
      parend
    parend
  
```

#### Procedure E

```

parforallpixels in the sub-area E do
  parbegin
    parfor  $x = \frac{N}{2}$  to  $N$  do
      for  $y = 0$  to  $N - x$  do
        Pixele in the sub-area  $E$  are projected
        onto the segment  $\alpha\beta_3$  of the diagonal
        with  $135^\circ$  along the direction of  $90^\circ$  ;
      parend
    parend
  
```

#### Procedure F

```

parforallpixels in the sub-area F do
  parbegin
    parfor  $y = 0$  to  $\frac{N}{2}$  do
      for  $x = N - y$  to  $N$  do
        Pixele in the sub-area  $F$  are projected
        onto the segment  $\alpha\beta_3$  of the diagonal
        with  $135^\circ$  along the direction of  $180^\circ$  ;
      parend
    parend
  
```

#### Procedure G

```

parforallpixels in the sub-area G do
  parbegin
    parfor  $y = \frac{N}{2}$  to  $N$  do
      for  $x = y$  to  $N$  do
        Pixele in the sub-area  $G$  are projected
        onto the segment  $\alpha\beta_4$  of the diagonal
        with  $45^\circ$  along the direction of  $180^\circ$  ;
      parend
    parend
  
```

#### Procedure H

```

parforallpixels in the sub-area H do
  parbegin
    parfor  $x = \frac{N}{2}$  to  $N$  do
      for  $y = x$  to  $N$  do
        Pixele in the sub-area  $H$  are projected
        onto the segment  $\alpha\beta_4$  of the diagonal
        with  $45^\circ$  along the direction of  $270^\circ$  ;
      parend
    parend
  
```

### 5 Evaluation of RPT Approach

The features from the RPT's have been used to recognizing a large set of characters which contain a lot of compound patterns. The features have been sent to the ISOETRP classifier [10]. A positive result has been achieved in an experiment of recognizing Chinese characters. The recognition rate can be shown in Table 1.

number of characters	1000	2000	3000
error rate	0.08%	0.10%	0.18%
rejection rate	0.12%	0.29%	0.54%
recognition rate	99.80%	99.46%	99.28%

Table 1. Recognition results

### 6 Conclusion

Regional projection transformation has a significant function which can transform a multiple-contour pattern into a single-contour pattern. This transformation also possesses parallelism which makes sense to develop a parallel algorithm to speed up the process of recognition. The proposed method in this paper can be used not only to recognize characters but also other patterns. VLSI implementation for parallel RPT algorithm has been achieved in our other work [9].

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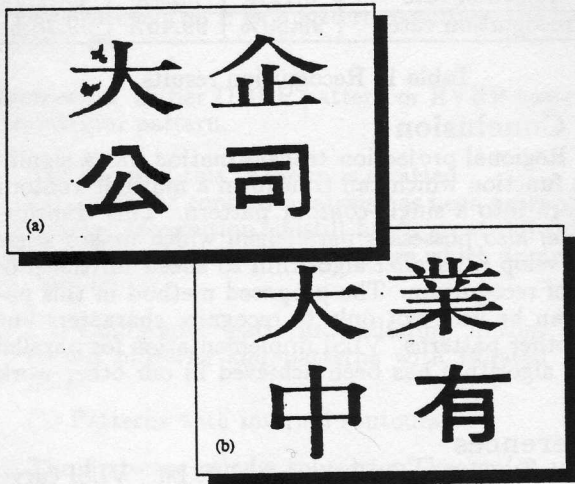


Fig. 1 Examples of patterns  
 (a) compound patterns  
 (b) integral patterns

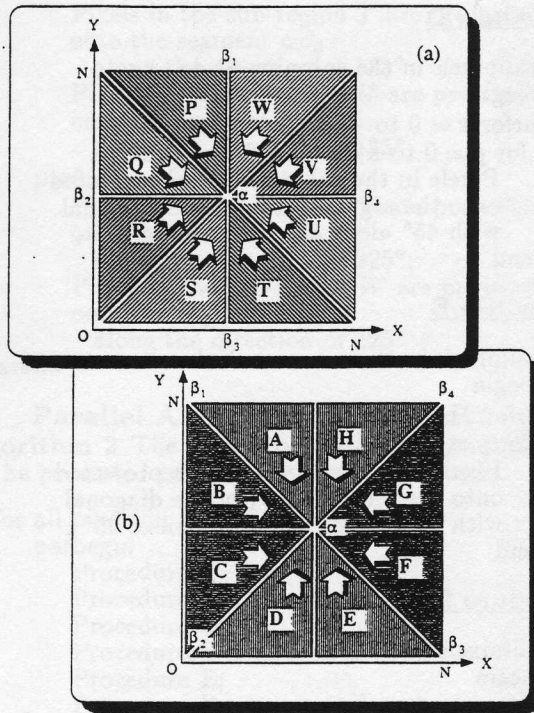


Fig. 2 Regional projection for  
 (a) HVRPT  
 (b) DDRPT

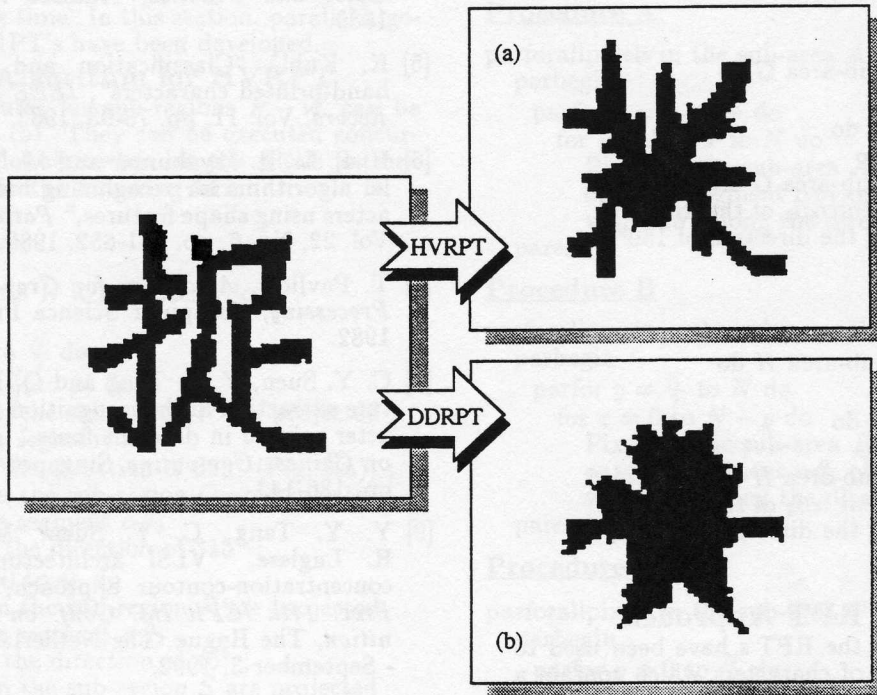


Fig. 3 Patterns transformed from regional projection transformation  
 (a) HVRP pattern  
 (b) DDRP pattern