Sampling Techniques in the Wiener Image Restoration

R.I.Campeanu

York University, Glendon College,
Department of Computer Science,
Toronto, Canada M4N 3M6

Abstract

This paper presents approximations to the Wiener filter based on both noise and signal sampling. When applied in the noise reduction problem, sampling substantially reduces the computing time, while in many cases still giving reliable results. The fastest method uses a spatial convolution filter obtained from the Wiener filter by means of hanning windowing.

1. Introduction

Noise reduction is one of the restoration processes for which a large number of studies were conducted. Most of these studies were performed in situations where the statistical characterization of the noise is known and can be described in some parametric form such as Gaussian. Another simple case is the coherent noise in the form of a two dimensional sinusoidal interference pattern superimposed on an image. This corresponds to a pair of symmetric impulses in a Fourier spectrum. The processing of such an image corresponds to applying two bandreject filters of radius one at the location of the impulses and then taking the inverse Fourier transform of the result.

In many examples however, the noise is signal independent and uncorrelated and cannot be represented by any analytical formula. The Fourier spectrum of the image contains a complicated pattern and any filtering of certain frequencies may eliminate much of the image information. Because of this, the processing is complicated and the result is always only an approximation of the true information.

Fortunately there are cases, such as those encountered in surveillance work, when the scene of interest may have many well known objects which can help to calibrate the signal. In many cases one can easily find samples of "pure" noise. These samples can correspond for example to a wall, window, or a portion of the sky uniformly lighted. The interactive restoration methods use such samples in order to eliminate the noise in the whole image, including regions where the objects of interest are not known.

Samples can be also taken to represent the signal. In some cases [1] the segmenting of the image into blocks was motivated by the fact that the image statistics change over its various regions. In addition to this, because in the Fourier transform complexity grows exponentially with the size of the image, the filtering of small blocks is the only way to perform the restoration of an image containing hundreds of thousands of pels on a slow machine. As much of the image processing work is now performed on Personal Computers, there is a demand for methods which do not involve excessive computing.

The work on new approximations to the restoration problem is also justified by a more general observation. Although each restoration technique claims optimality with respect to different criteria, it should be noted that a degraded image suffers a fundamental loss of information, compared to the image which would have resulted in the absence of the degradation. The various methods proposed so far cannot recover this fundamental loss. Since the state of art does not allow an accurate understanding and mathematical modelling of the human visual process, it is not yet possible to define precisely the meaning of optimum restoration [2]. In the absence of a general formulation of the image restoration problem which would take into account observer preferences and capabilities, the interactive restoration is a very powerful methodology.

Our paper will present various levels of approximation in the Wiener filtering of noise. It will show how the new levels of approximation will decrease the computing time, while the quality of the restored image will not necessarily deteriorate. We shall also show that the Wiener approximations are superior to the simple smoothing techniques.

2. The Wiener Filter.

One of the most often employed interactive restoration methods is the Wiener filter. The original form of this filter in the frequency domain is [3]:

$$H(u,v) = \frac{|F(u,v)|^2}{|F(u,v)|^2 + \dfrac{S_n(u,v)}{S_f(u,v)}}$$

where $F(u,v)$ is the Fourier spectrum of the whole image and $S_f(u,v)$ and $S_n(u,v)$ are the signal and noise spectral densities. As the spectral densities are often unknown, there ratio is usually replaced by a parameter K, which can be varied to obtain the best noise reduction:

$$H(u,v) = \frac{|F(u,v)|^2}{|F(u,v)|^2 + K}$$

Ekstrom [4] showed that K must satisfy a certain equation, while other authors [5] preferred to determine K by successive iterations. Harries [5] obtained good results by taking the value of this parameter as some multiple of the noise variance.

Our approach is based on Harries' work. We manually select a noise sample by positioning a 32x32 pel frame on an area of "pure" noise. Our images are 8-bit 640x480 pel grey. The noise sample is calibrated by subtracting the sample average. The value of K is then taken as:

$$K = 2N\sigma$$

where N is the number of pixels in the noise sample and σ is the value of the noise statistical variance.

The parametric Wiener filter still implies that one calculates the Fourier spectrum of the whole image, which on a microcomputer can be extremely time consuming. The next section will present a few methods to overcome this problem.

3. Image Sampling

A first level of approximation is to divide the whole image in 20 x 15 blocks, each of 32x32 pixels. The filter H(u,v), different for each sample, is multiplied with F(u,v) and the result processed by an inverse Fourier transform. We shall call this approximation the Block-based Wiener (BW) filter.

We also tried an alternate formula, which exploits the fact that both noise and signal samples are of the same size (32x32 pel):

$$H(u,v) = \frac{|F(u,v)|^2 - |N(u,v)|^2}{|F(u,v)|^2}$$

with H(u,v)=0, when the value given by this equation is negative. This method, using N(u,v) and F(u,v) as the discrete Fourier transforms of the noise and signal samples, proved to be less reliable than BW and was not employed in the next levels of approximation.

A further level of approximation would be to employ in the Wiener equation the discrete Fourier transform of a "representative" signal sample. This implies that one produces a unique H(u,v), which can then be applied on the 300 blocks. By moving the calculation of the Wiener filter out of the blocks loop, one gains some computing time relative to BW. We call this approximation the Representative Sample-based Wiener (RSW) filter.

Choosing a "representative" signal sample is not an easy task. The challenge is to represent the whole range of frequencies of an image in a 32x32 complex array F(u,v).

We tried 3 solutions:
- calculating the signal sample by adding together 10 blocks randomly distributed (filter RSW1).
- averaging the 10 blocks randomly distributed (filter RSW2).
- manually positioning a 32x32 pel frame on a representative area i.e. an area which contains a wide variety of frequencies (filter RSW3).

The last level of approximation was based on RSW. Instead of applying this filter to the 20 x 15 image blocks, we built a FIR filter that approximates H(u,v). One of the simplest ways to calculate a FIR filter is by applying a windowing technique, such as the hanning method [7]. For a two-dimensional problem the 3 x 3 hanning window is given by:

$$H_h(x,y) = 0.25\, H(x,y)$$

$$[1+\cos(2\pi x/3)][1+\cos(2\pi y/3)]$$

where x and y have the values -1, 0, and 1.

This spatial convolution kernel is then applied to the whole image. The computing time for this approximation, which we call the 3 x 3 Kernel based on the RSW filter (K3-RSW), is expected to be substantially smaller than in the other methods.

4. Results

Our work was performed on a PS/2 Model 50 (with math coprocessor) running DOS. The processing of a 640 x 480 pel image required between 2 and 10 CPU minutes depending on the approximation employed. The most expensive model was BW, the RSW filters only a little faster than BW, and the cheapest method K3-RSW.

Figure 1 presents the image degraded by additive noise. From the results obtained with the models employed in this work we have selected two, shown in Figures 2 and 3.

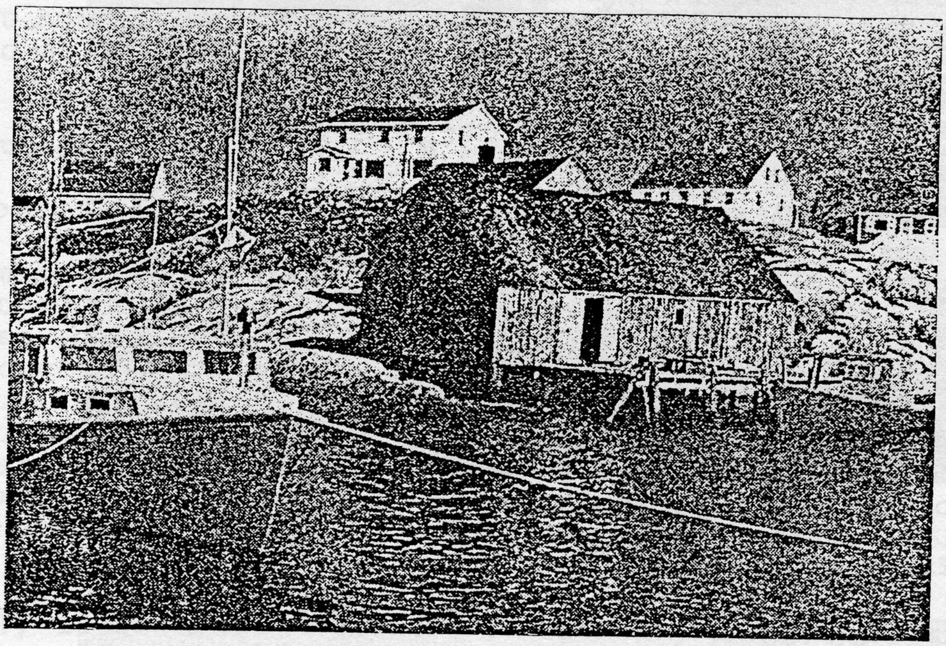

Figure 1: The test image degraded by additive noise

Figure 2 presents the result of applying the filter RSW1. As already explained, this model calculates the signal sample by adding together 10 blocks randomly distributed, while the noise sample is chosen manually from the area denoted in Figure 2 by N. The obvious failure of this approach is explained by the difficulty in building a representative signal sample by adding or by averaging random blocks.

Both operations modify the original distribution of frequencies.

A much better approach is RSW3, in which the signal sample is manually chosen from an area with a rich spectrum of frequencies. Figure 3 shows the result obtained with this model. We have denoted with S the area from where the signal sample was taken, while N denotes the area from which the noise sample was extracted.

The fastest approximation was K3-RSW. This method lead to the following kernel:

0.004067	0.037728	0.021259
0.034116	0.529133	0.136463
0.021259	0.150911	0.065065

These numbers clearly indicate a smoothing filter kernel. It is an asymmetrical filter, with the most important contributions coming from the neighbouring

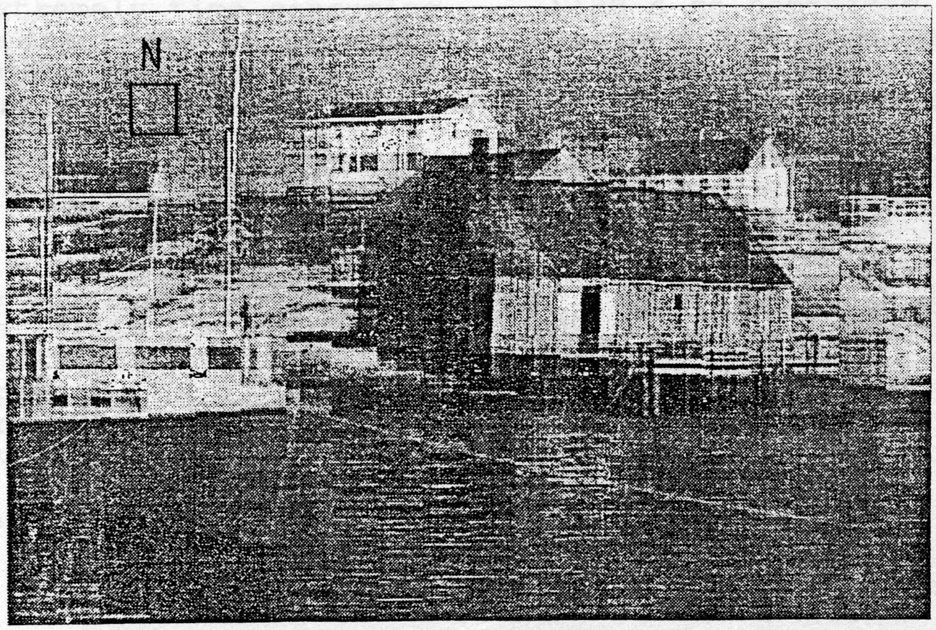

Figure 2: The result of applying the filter RSW1.

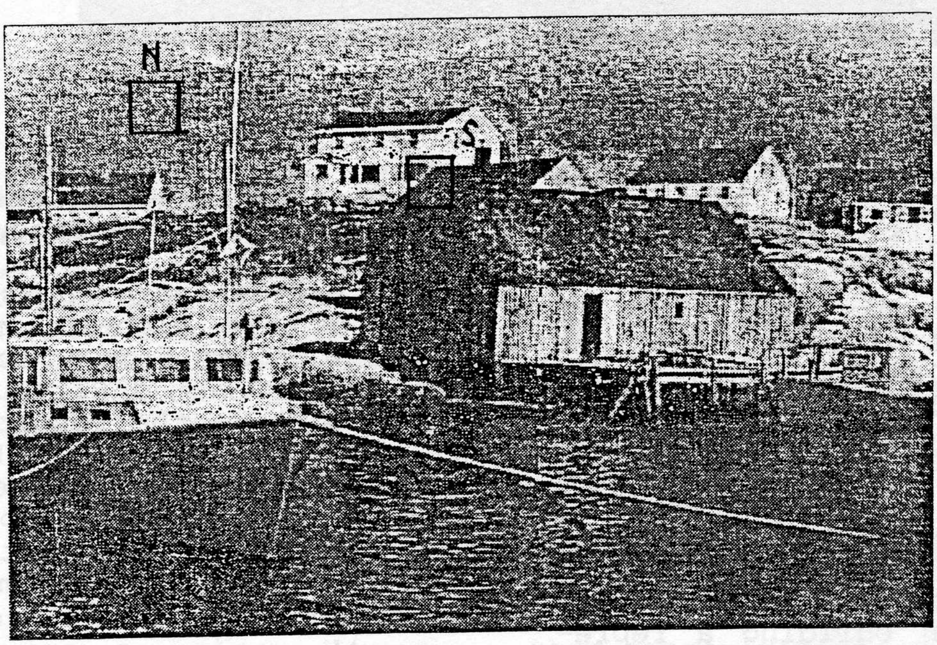

Figure 3: The result of applying the filter RSW3.

pixels below and to the right. The result of this filter is comparable with that of Figure 3. The plus that one gets with K3-RSW3 is the absence of any additional speckles. The result of K3-RSW is clearly superior to the results obtained with other symmetrical smoothing kernels. Images corresponding to the various filters mentioned in this paper but not shown here, will be presented at the conference.

5. Conclusions

This paper is the result of a study on low-cost approximations of the image restoration problem. We developed a number of sampling techniques which were used in the noise reduction of a grey image. The gradual increase of the level of approximation allows for a good understanding of the effects of various changes designed to reduce the computing time.

Working on a microcomputer on relatively large images (640x480 pel images) we obtained good results at a reasonable computing cost. The best two approximations were the Blocks-based method (BW) and the spatial convolution kernel produced by K3-RSW. The image produced by BW is slightly crisper, but the image produced by K3-RSW presents no additional speckles. If one adds that K3-RSW is at least 5 times faster than BW, we could conclude that this approximation is probably the most attractive result of our study.

References

1. Ingle V.K., Radpour A., Woods J.W., and Kaufman H., "Recursive Estimation with Non-Homogeneous Image Models", Proc. IEEE Patern Recognition and Image Processing, Chicago, pp.105-8 (1978)

2. Venetsanopoulos A.N.,"Digital Image Processing and Analysis", Proc. Summer School on Signal Processing, Les Houches 1985 (J.L.Lacoume, T.S.Durrani and R.Stora, eds.), Elsevier (1987)

3. Gonzalez, R.C. and Wintz, P., "Digital Image Processing", Addison-Wesley, pp.209 (1987)

4. Ekstrom, M.P., "Digital Image Processing Techniques", Academic Press (1984)

5. Harries, J.L. , "Potential and Limitations of Techniques for Processing Linear Motion-Degraded Images", in "Evaluation of Motion Degraded Images", NASA SP-193, pp.131-8 (1968)

6. Karayiannis N.B. and Venetsanopoulos A.N.,"Image Restoration: A Regularization Approach", Proc. 3rd Intl.Conf.on Image Processing and its Applications, IEEE, New York, pp.1-5 (1986)

7. Rabiner, L.R. and Gold, B., "Theory and Application of Digital Signal Processing", Prentice-Hall, pp.92 (1975)