Stereo Matching
Based on Fixed Point Theorem

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Abstract

The main problem in stereo vision is to find corresponding points in left and right images. The method of computing disparity proposed in this paper is based on a fundamental topological theorem: for any continuous transformation of a disk to itself there is at least one fixed point. Locating the fixed point in the mapping of the image intensity function between left and right images gives the disparity at the location of the fixed point. The sum of squared intensity differences within a window is used to find the fixed point.

Key words: *Stereo Vision, Disparity, 3D Vision, Computer Vision, Fixed Point.*

Résumé

Le principal problème en vision stéréo est de trouver la correspondance de points entre l'image de gauche et l'image de droite. La méthode de calcul de disparité proposée dans ce rapport est basée sur un théoreme fondamental de topologie: pour toute transformation continue d'un disque sur lui-même, il existe au moins un point fixe. La position du point fixe dans la transformation de la fonction d'intensité de l'image de droite et l'image de gauche donne la disparity au point fixe. La somme des différences d'intensité au carré à l'intérieur d'une fenêtre est utilisée pour trouver le point fixe.

Mots clés: *Vision stéréo, disparité, vision 3D, visionique, point fixe.*

1 Introduction

In stereo vision the relative depths of objects in a scene can be calculated by simple triangulation if disparity is known. The hardest part of this method is to identify corresponding points in both images [1].

The method proposed in this paper is an efficient way of finding disparity between pixels in left and right images. The method is based on a fundamental topology theorem introduced in 1910 by L.E.J. Brouwer [4]. Brouwer proved that any continuous function from a non-empty, convex, compact subset of R^n into itself has at least one fixed point. Using this theorem, the disparity between two windows can be calculated by locating a fixed point in a mapping of a left image onto a right image.

Brouwer's Theorem.

Let C be a nonempty set in the n-dimensional Euclidean space R^n. Then for any continuous function f from C into itself there exists at least one point p^* such that $f(p^*) = p^*$, i.e. the function has at least one fixed point [4].

One of the simplest ways in which a disk may be mapped to itself is by rotation about its center. This is a rigid transformation preserving the topological properties of the disc. The transformation is continuous, where each point p of the disk is mapped to some unique point p', which is the image of no other point [2]. It is clear that for a rotation about a centre point, for a rotation angle ϕ which is not an integer multiple of 2π, there is only one point which maps to itself, namely the centre of the disk [2].

Application to stereo vision.

The method based on Brouwer's theorem can be applied to stereo matching if both left and right images have been taken with similar cameras separated by a baseline. The left and right images (or small windows within these images) ideally would differ only by translation along horizontal X axis and otherwise be identical (in practice other differences may be present due to perspective projection distortions and illumination differences).

Let us consider the rotated left image as a mapping of the right image onto itself. This mapping can be represented by translation (due to disparity, assumed constant within a small window) and rotation. According to Brouwer's theorem there are at least two points, one in the rotated left image and the other in the right image, having the same coordinates and the same intensity values, i.e. the fixed point.

To derive the coordinates of the fixed point in coordinate system centered at a pixel where disparity is being measured let us consider transformation of coordinates involving translation (by vector x_0, y_0) and rotation (by angle θ) defined by

$$x' = (x - x_0) \cdot cos\,\theta + (y - y_0) \cdot sin\,\theta$$
$$y' = (y - y_0) \cdot cos\,\theta - (x - x_0) \cdot sin\,\theta.$$

If the image is rotated about a point on the epipolar line then $y_0 = 0$ and the formulas become

$$x' = (x - dx) \cdot \cos\theta + y \cdot \sin\theta \tag{1}$$

$$y' = y \cdot \cos\theta - (x - dx) \cdot \sin\theta \tag{2}$$

where dx is the disparity. Since at the fixed point we have $f(p^*) = p^*$ then

$$x' = x \tag{3}$$

$$\text{and} \quad y' = y \tag{4}$$

Combining equations (1),(3) and (2),(4) gives

$$\begin{aligned} x &= x \cdot \cos\theta + y \cdot \sin\theta - dx \cdot \cos\theta \\ y &= y \cdot \cos\theta + dx \cdot \sin\theta - x \cdot \sin\theta \end{aligned}$$

Solving for x:

$$(1 - \cos\theta) \cdot x = y \cdot \sin\theta - dx \cdot \cos\theta$$

$$x = \frac{dx}{2} \tag{5}$$

Solving for y:

$$\begin{aligned} (1 - \cos\theta) \cdot y &= dx \cdot \sin\theta - x \cdot \sin\theta \\ (1 - \cos\theta) \cdot y &= dx \cdot \sin\theta - \frac{dx}{2} \cdot \sin\theta \\ y &= \frac{\frac{dx}{2} \cdot \sin\theta}{1 - \cos\theta} \end{aligned}$$

$$y = \frac{\frac{dx}{2}}{\tan\frac{\theta}{2}} \tag{6}$$

Therefore, the disparity between left and right images is

$$\begin{aligned} dx &= 2x \\ \text{or} \quad dx &= 2y \cdot \tan\left(\frac{\theta}{2}\right) \end{aligned}$$

where x and y are the coordinates of the fixed point in the coordinate system centered at the rotation point.

Finding the fixed point in digital images.

Since digital images are used, the intensity function is discrete and pixels coordinates are rounded off after rotation. Because of this, some pixels surrounding the fixed point are likely to be mapped onto themselves. The size and shape of this region is determined by the rotation angle. For a small rotation angle the region will be large and for a rotation angle greater than 30 degrees only the centre point will map onto itself.

For perfectly identical left and right images, the differences of intensities within the window corresponding to the mapping of pixels onto themselves for a given angle is zero. In practical cases, when the left and right images are not identical, the sum of square differences (SSD) over a small mask is used for matching pixels in the fixed point region. The SSD is calculated at all possible fixed point locations and the point with minimal value is used as the fixed point location. The possible fixed point locations for each disparity value can be found by using formulas (5) and (6).

Since it was assumed that all points in the left image were translated uniformly, these formulas are based on the assumption that disparity is constant. In practice disparity varies within a window and the detected value dx is only applicable to the pixel at the location of the fixed point. Since the fixed point can be located anywhere inside the window (not necessarily in the centre), scanning the entire image may leave some points in the image with no disparity values assigned.

Experimental Results.

To determine the disparity at a pixel the left image was rotated by 90^o counterclockwise about this pixel. The SSD for a mask size of 7 by 7 pixels was calculated for all potential fixed points within a certain range. For simplicity, a maximum disparity value was assumed, limiting the size of the region to be searched. The pixel with the minimal value of SSD (i.e. fixed point) was then assigned the corresponding disparity.

Figure 1 represents a simulated left view of a stereo image pair. This view is a reproduction of the right view with introduced disparity regions (disparities: 2, 4 and 8 pixels).

Figure 3 shows a left image of an actual scene with objects at different distances.

Figures 2 and 4 represent disparities calculated using the fixed point method for simulated and actual stereo image pairs. Each colour represents a different disparity value. The actual distances can be associated with detected disparities using the stereo imaging system parameters (focal length and baseline).

Conclusion.

The stereo matching algorithm based on Brouwer's theorem presented in this paper can be used to measure disparities in stereo images.

References

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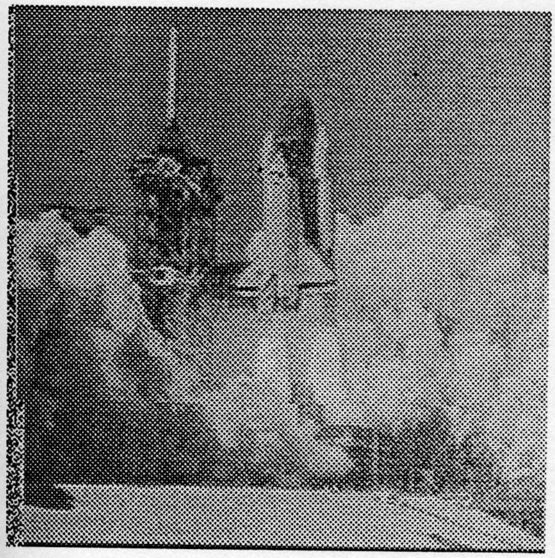

Figure 1: Left view of simulated 3D scene.

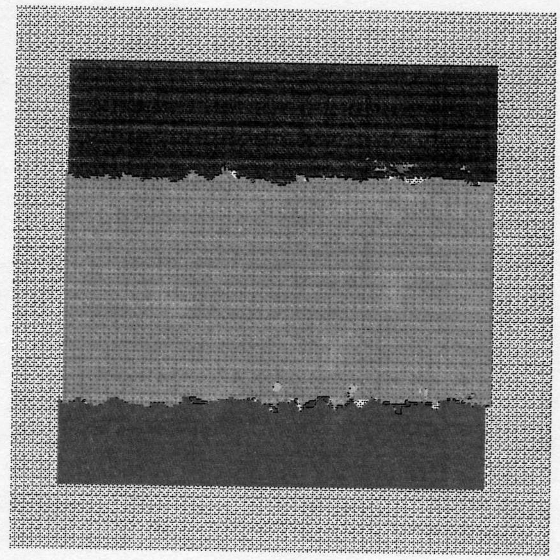

Figure 2: Disparities of simulated 3D scene.

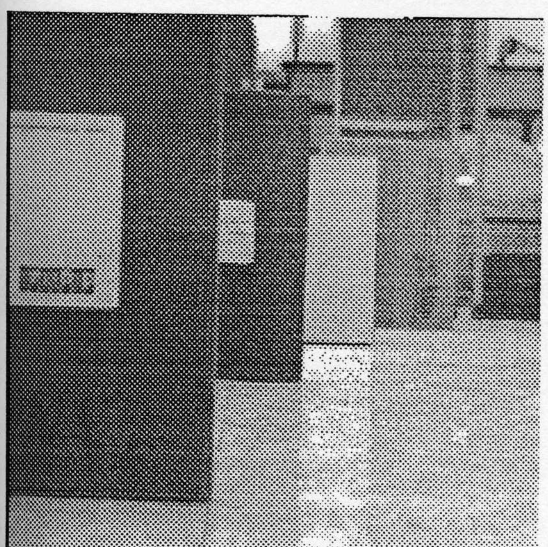

Figure 3: Left view of 3D scene.

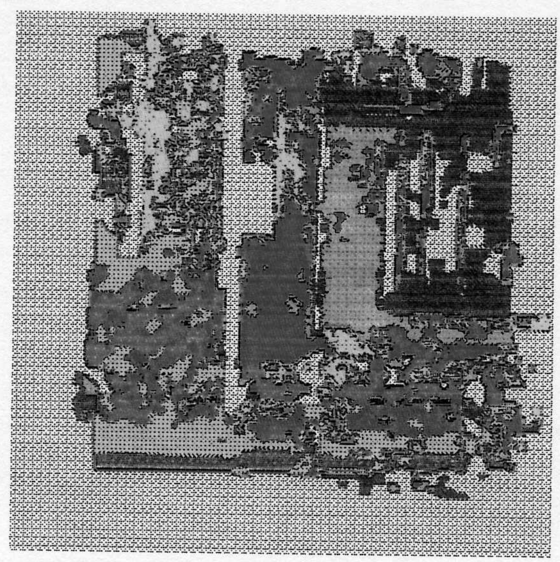

Figure 4: Disparities of 3D scene.