

Efficient Image Registration by Intensity Combinatorial Minimization

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Abstract

In this paper a new registration method is introduced. The *Intensity Combinatorial Minimization Method (ICMM)*, finds the registration point of a template and an image by finding the point that minimizes the number of grayscale intensity combinations between the template and the image. The method has several advantages over existing methods. (1) The method is computationally efficient- only simple calculations are required. (2) The method is invariant to the image (or the template) processed by an injective (one-to-one mapping) function. (3) The method produces good overall registration results. The method also promises to be suitable for multi-spectral image registration.

1. Introduction

The problem of locating a template in an image is a central problem in computer vision. It arises frequently in areas of scene analysis such as multi-spectral image analysis [1] and object recognition [2]. It is given the term image registration, when two images (or more) of the same scene need to be aligned, as in multi-spectral image analysis. When one wants to locate a certain pattern (template) in a given image, as in object recognition, the term template matching is used. In such a case the location of the template with respect to the image in terms of translational, rotational and scaling offsets is of interest. In this paper the emphasis is on the image registration problem rather than the template matching problem, although the method presented here can be applied to a large extent to the template matching problem.

We present a new registration method, termed the *Intensity Combinatorial Minimization Method (ICMM)*, that has several advantageous features over existing methods. In addition to being computationally efficient, *ICMM* has good performance, is not very noise sensitive and is invariant to *Injective Functions*, a particular class of image processing functions.

2. Image Registration Methods

Area correlation is one of the earliest and most widely used method for image registration [3]. Given a template T , of dimension $K \times L$ and an image I of dimension $M \times N$, the *area correlation* of the two images is computed by using the cross-correlation of the two images,

$$R(m, n) = \sum_{k=1}^K \sum_{l=1}^L T(k, l) \cdot I(m+k, n+l) \quad (1)$$

where $1 \leq m \leq M - K + 1$, $1 \leq n \leq N - L + 1$.

The location (m^*, n^*) where the maximum occurs, is taken as the translational registration offset. Since the maximum value of R found will not necessarily yield the best registration point, the cross-correlation needs to be further normalized. Equation (1) after normalization becomes,

$$R(m, n)^2 = \frac{\left(\sum_{k=1}^K \sum_{l=1}^L T(k, l) \cdot I(m+k, n+l) \right)^2}{\sum_{k=1}^K \sum_{l=1}^L (T(k, l))^2 \cdot \sum_{k=1}^K \sum_{l=1}^L (I(m+k, n+l))^2} \quad (2)$$

As before, where R attains a maximum, the point is taken as the point of optimum registration. Although this method, the *Normalized Area Correlation Method (NACM)*, is quite robust for most applications, its main disadvantage is that it is computationally intensive. To calculate R , for I and T of the indicated sizes, requires the order of $M \times N \times K \times L$ multiplication's. Even with the use of an FFT algorithm, the expense of the calculation remains the same, since only the numerator of (2) can be calculated using an FFT.

Another way to register images is to subtract the template from the image and observe the result. This is what the *Sum of Absolute Difference Method (SADM)* [4] does,

$$R(m, n) = \sum_{k=1}^K \sum_{l=1}^L |T(k, l) - I(m+k, n+l)| \quad (3)$$

The resulting matrix R is then searched for the minimum which is taken as the point of registration.

$SADM$ is not as robust as $NACM$, but is more efficient. For the same image I and template T , $M \times N \times K \times L$ additions (subtraction's) are required, which is not as costly as the multiplication's required by $NACM$. Other variations of $SADM$ exist, such as taking the square root instead of the absolute value in equation (3) (a least square method), a somewhat more costly variation of $SADM$.

The large computational cost and marginal performance of these and other methods has prompted much research into developing other registration methods. A recent survey of image registration techniques can be found in [5].

2.1 Point Matching Registration Methods

Point matching is one method of registration that reduces the computational cost significantly. Here, a set of points in the images are extracted and correspondence between the points is found. The points extracted are usually distinct image feature points. O'Gorman [6] and Bose [7] locate the centroids of fiducials in the images and use those points for registration. Another point based method that matches a pattern of points are described in [8]. Here a simple approach to point matching is performed by measuring the displacement of feature points and counting the number of displacement occurrences. In [9], a relaxation point pattern matching method is used. A figure of merit is assigned to each pair of matched points according to how closely other pairs in the set match. Wang et al. [10] extended the method of [9] to allow matches under position and orientation changes. Stockman, et al. [11] used a clustering approach to match image features between the image and the model, based on local evidence. Goshtasby [12] used normalized invariant moments to match images that are offset rotationally. Goshtasby also proposed another method in [13] using points laying on the convex hull of a set. A region based approach was later presented by Goshtasby in [14]. Cox et al. [15] presented an iterative procedure matching line segments to find correspondences between the image and the template.

3. Problems with Classical Registration Methods

Both $NACM$ and $SADM$ are very sensitive to noise in the image. These difficulties can be seen by the following simple example. Let the template be T , and the image being matched to is the image I , shown in Figure 1. I consists of 3 sub-images, I_1 , I_2 , and I_3 . Clearly, by comparing the template with the sub-images, one's visual perception of the templates and the sub-images, concludes that sub-image I_1 resembles the template the most.

Table 1 shows the results found using $NACM$ and $SADM$. $NACM$ detected sub-image I_2 as the best match. $SADM$ resulted in both I_2 and I_3 as the best match. Clearly both methods failed to find the correct result, I_1 .

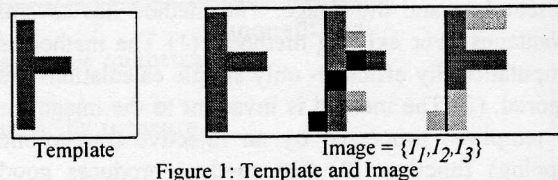


Table 1: Matching Results.

Method	I_1	I_2	I_3	Best Match
$NACM$	0.966	0.996	0.992	I_2
$SADM$	230	229	229	I_2 / I_3
$ICMM$	3	16	5	I_1

The incorrect results obtained by these methods can be attributed to the criteria that is used to find a match as given by their respective equations. For $NACM$, equation (2) calculates R by the cross correlation of the template and the image window as it is being swept. The presence of noise, even at only one point, if large enough, (e.g. image I_1) will be sufficient to affect the outcome of R at that point and hence throw off the complete matching process. A similar argument also applies for $SADM$ and equation (3). However, in this case the effect is somewhat less pronounced since the presence of noise is not magnified through multiplication as is done for $NACM$. It is this sensitivity to noise which leads to inaccurate registration results using these methods. Some of the other inconveniences associated with both $NACM$ and $SADM$:

- Both are sensitive to images processed by image processing functions. Once an image is processed by some of these function (e.g. histogram equalization, image inversion) it is very difficult to

locate the template in the image or register the images using either of these two methods.

- Both require an enormous amount of time when the images are large, the order of computations required are of the order $O(n^4)$.
- Both do not perform well for multi-spectral and color images.

The *Intensity Combinatorial Minimization Method (ICMM)*, overcomes many of these deficiencies to a great extent. In addition to being less noise sensitive and invariant to a particular class of image processing functions, the method is computationally efficient and has produced good registration results, as we show next.

4. The Intensity Combinatorial Minimization Method (ICMM)

The *Intensity Combinatorial Minimization Method (ICMM)* is a registration method based on a new metric for detecting similarity between images. The similarity measure is based on the simple fact that images of the same scene should appear similar. This implies that if an intensity of a point (grayscale value) of an object in one scene changes to a different value in another scene, then all points that are at the same intensity (or most of them) should also map into the same new value (or a finite set of possible grayscale values). In other words, the variation of the intensity of all points with a certain grayscale value in the first image to that of the second image should be small.

ICMM differs from other (control) point methods in that all pixels of the images are taken as control points. In contrast, most other control point methods, employ some form of calculation [8], or require the image to be pre-processed by some process to extract a subset of points and may require high level feature extraction [11], or both.

4.1 Formal Definition

ICMM locates a template match by sweeping the template over the image, and examining the overlaid pixels to calculate the registration matrix R . This is done by counting the number of **distinct combination pairs** of the intensity of the template, and the image window. R is then searched for the minimum value which corresponds to the registration and match point. Theoretical derivations can be found in [16].

ICMM is formally defined as,

$$R(m,n) = \sum_{k=1}^K \sum_{l=1}^L \Gamma(T(k,l), I(m+k, n+l)) \quad (4)$$

where, initially $S = \phi$, and

$$\Gamma(x,y) = \begin{cases} \text{if } (x,y) \notin S, & \Gamma(x,y) = 1, (x,y) \in S \\ \text{if } (x,y) \in S, & \Gamma(x,y) = 0 \end{cases} \quad (5)$$

$$\text{Registration point } (m^*,n^*) = \{(m,n) \mid \min\{R(m,n)\}\} \\ \text{for } 1 \leq m \leq M-K+1, 1 \leq n \leq N-L+1 \quad (6)$$

The value of R at any point (m,n) is bounded by a lower and an upper bound, $\ell_1 \leq R \leq \ell_2$. The lower bound, ℓ_1 is the number of distinct grayscale values in the template, which can not exceed 2^n , where n is the number of bits used to represent the image. The upper bound, ℓ_2 , is the size of the template ($K \times L$), or the entire number of possible combinations, 2^{2n} , whichever is smaller.

4.2 The Grayscale Combinatorial Plot

If we plot the grayscale combination pair set, S , of equation (5) on a two dimensional plot, we find the results useful in conveying information about the template and the image. We call this plot the *Grayscale Combinatorial Plot (GCP)*. In this plot, the x-axis represents the grayscale value of the template and the y-axis represents the grayscale value of the image. The number of points in the plot, is bounded by the same bounds, ℓ_1 and ℓ_2 that bound R .

Figure 2 shows the *GCP* at three different locations close to the correct registration point¹, (m^*,n^*) , for the *phone* image of Figure 3. As the image point of correct registration is approached the random distribution of the *GCP* becomes more ordered, and at the correct registration point, the distribution of the points collapse into a straight diagonal line, which implies that every grayscale value of the template matches to

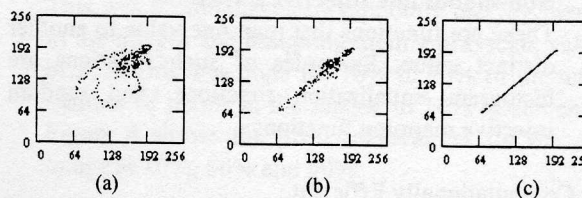


Figure 2: GCP for the *Phone* image close to the correct registration point (m^*,n^*) ; (a) at (m^*-1, n^*-1) (b) at (m^*, n^*-1) (c) at (m^*, n^*) .

¹ Here the template is a subregion of the original image

the same grayscale value of the image. This is, in essence what *ICMM* does, minimize the number of (distinct) points in the *GCP*.

4.3 Features of the ICMM

ICMM exhibits several appealing features for registration applications:

1. Lower Sensitivity to Noise.

From equation (4), only the number of grayscale combination pairs between the image and the template affects the outcome of *R*. No weight is given to the difference in pixel intensity values between the template and the image windows. As a result, this method's sensitivity to noise is less than other methods (e.g. *NACM* and *SADM*). The presence of several points of noise does not disturb the outcome of *R* greatly.

The results of applying *ICMM* to our previous example of Figure 1, is also given in Table 1. *ICMM* correctly identified sub-image I_j as the correct match to *T* in *I*.

2. Invariance to Injective Functions

ICMM is invariant to images processed by injective functions [17]. Examples of injective functions are:

- **Monotonically increasing functions**
Examples of monotonically increasing functions include image contrast functions such as $(x/L)^a$ and $a \cdot x + b$ ($a \neq 0, a > 0$). Here x is the grayscale value at any image point, $L = 2^n$ (n is the number of bits used to represent the image), and a is a constant.
- **Monotonically decreasing functions**
Examples of monotonically decreasing functions include inverting the image; $(L-x)$, and decreasing contrast functions such as $a \cdot x + b$ ($a \neq 0, a < 0$) where a is a constant.
- **Non-monotonic Injective Functions**
These are functions that map one value to another distinct value. Examples of such functions are histogram equalization functions, and random injective mapping functions.

3. Computationally Efficient

ICMM is computationally efficient since no image calculations are required. Other methods require the order of $O(n^4)$ algebraic computations to search the

image for a match. The only overhead associated with the method is the extra storage cells required to store the different combinations found. The maximum number of cells required is $\ell_1 \times \tau_1$, where ℓ_1 is the number of distinct grayscale values in the template and τ_1 is the number of distinct grayscale values in the image.

4. Suitable for Registering Multi-Spectral Images

Registering multi-spectral images is a more difficult problem than registering grayscale images. Using registration methods that compare the grayscale values of image pixels (e.g. *NACM* and *SADM*) produces incorrect results since these methods register images by minimizing the difference between grayscale values for the images being registered.

Since *ICMM* performs registration by minimizing the number of possible variations of grayscale values between the two images, it can be used for multi-spectral image registration. Although, the method will not always produce the correct result (see below), it is more likely to produce less erroneous results than methods that simply depend on comparing the grayscale values of the images.

5. Use of Smaller Template Sizes

A smaller template can be used with *ICMM*. Other methods may fail to register images when smaller templates are used [18].

4.4 Limitations of ICMM

The performance of *ICMM* degrades as the number of bits used to represent the image decreases. This is because the method relies on matching a certain pattern (the template) from a vast domain of possible patterns (see Theorems 1 and 2 in [16]). As a direct consequence, *ICMM* will not perform well for images that have a small number of gray level intensities such as binary images. For typical applications with 256 intensity levels the results are good, as we show below.

Also implied from the above, is that performance of the method degrades as the template size decreases, which is also true for all methods in general.

A more serious limitation of *ICMM*, is that it will not perform adequately for images with large regions of constant grayscale value. However, for such images, employing an additional criteria, the injectiveness criteria [16], will overcome this limitation.

The method as expressed by equation (4) will only register images that are offset by a translation. The

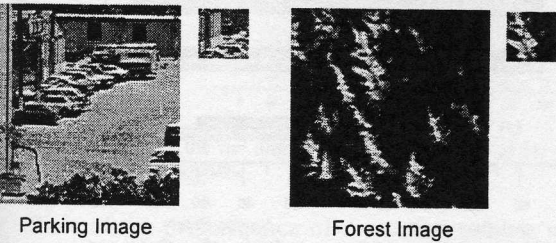
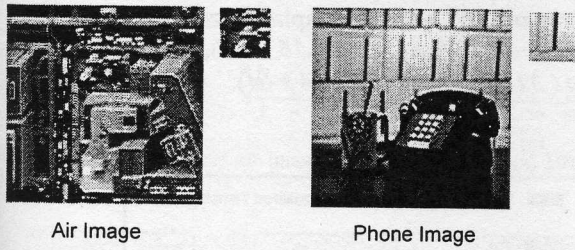


Figure 3: The image set shown with their templates.

method will not work for images that are offset by a rotation or by a scaling factor. Adapting the method to finding images that are offset by these parameters is currently being researched.

5. Experimental Results

Here we present the results of applying the *ICMM* to several images. First, the template matching criteria is evaluated to a set of images. Next the registration method is applied to register color images.

5.1 Template Matching Performance

The *ICMM* template matching criteria is compared with other registration methods to evaluate its performance with respect to:

- Noise addition
- Information removal
- Template size
- Image processing using injective functions.
- Time required to find the match.

For these tests the images of Figure 3 were used. They consist of:

- (a) The *air* image, an aerial view of an urban area.
- (b) The *phone* image a scene containing a telephone.
- (c) The *parking* image, a scene of a parking lot.
- (d) The *forest* image, an aerial view of a forest.

All of the images are $128 \times 128 \times 8$ -bits, grayscale images. A sub-region of each image, 32×32 in size, (as shown in the Figure 3) was taken from each image

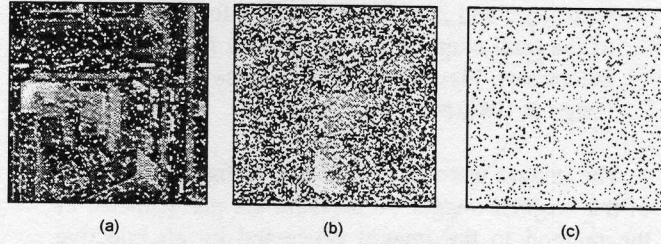


Figure 4: Pixel removal % for air image at (a) 10 (b) 50 (c) 90.

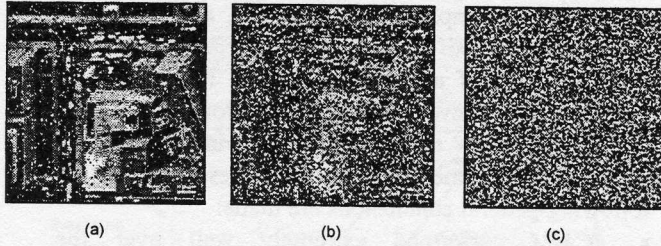


Figure 5: Noise % for air image at (a) 10 (b) 50 (c) 90.

and used as the template for registration. Some of the attributes of these templates are shown in Table 2.

Table 2: Template Properties

Image	Distinct Gray Levels	Entropy
<i>Air</i>	133	6.66
<i>Phone</i>	108	5.80
<i>Parking</i>	.148	6.75
<i>Forest</i>	115	6.28

5.1.1 Effects of Noise, Information Removal and Histogram Equalization on Template Matching

To test the performance of *ICMM* and its sensitivity to the presence of noise and to having parts of the images not present, the images were modified as follows,

1. Pixel Removal

Each image had parts of it randomly wiped out. This was accomplished by randomly picking pixels in the images and assigning them a grayscale value of 255. This was done for 10% to 90% of the total number of pixels in the image, in 10% increments. Figure 4 shows the pixel removal set for the *air* image at 10%, 50% and 90%.

2. Noise Addition

Random white noise was introduced to each image. This was accomplished by randomly picking pixels in the images and assigning them a grayscale value

between 0 and 255. This was done for 10% to 90% of the total number of pixels in the image, in 10% increments. Figure 5 shows the noise set for the *air* image at 10%, 50% and 90%.

The images were then registered using the templates of the original image. To measure the performance of the method to the images processed by an injective function, a simple injective function, histogram equalization, was applied to the templates of the images. Each template was equalized, and the above tests were repeated.

The results of applying *ICMM* to the images of Figure 3 are shown in Figures 6 through 9. These figures also show the results of applying *NACM* and *SADM* to the same images. From these figures, the following general conclusions are made:

- *ICMM* performed extremely well over the complete range of tests. It was able to find the correct registration point in every single case, for both the pixel removal set and the noise addition set. The results were the same for the equalized template case.
- *SADM* performed extremely well in the unequalized template case, but performed poorly in the equalized case, for both the pixel removal set and the noise addition set. This method also showed the most sensitivity to template equalization. (The only instance the performance did not degrade was for the *parking* image. This is due to the fact that equalizing this image template did not modify it significantly).
- *NACM* performed poorly in all cases. It is the most sensitive to noise or having parts of the image missing.

On average, using *ICMM* instead of *NACM* increased the range by 218% for pixel removal, and 100% for noise addition, for both the equalized and unequalized case. Using *ICMM* instead of *SADM* increased the range by a minimum of 5% for both pixel removal, and noise addition, for both the unequalized case, and increased the range by an average of 52% for the equalized case. The *GCP* for the air image at the registration point at 10%, 50% and 90% for pixel removal and noise addition is shown in figures 10 and 11, respectively.

5.1.2 Performance Evaluation as a Function of Template Size

Another set of experiments were conducted to observe the performance of the method as a function of

template size. The air image was used in this experiment and several template sizes were used as follows, 64 (8 x 8), 256 (16 x 16), 512 (32 x 16), 1024 (32 x 32), and 4096 (64 x 64).

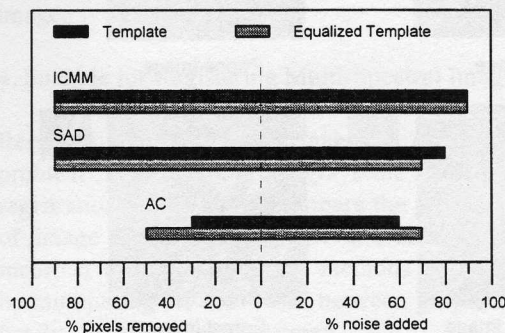


Figure 6: Registration results for the air image.

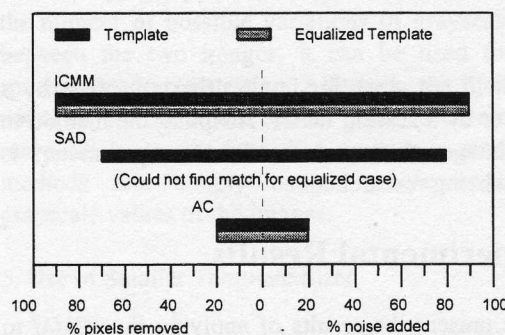


Figure 7: Registration results for the phone image.

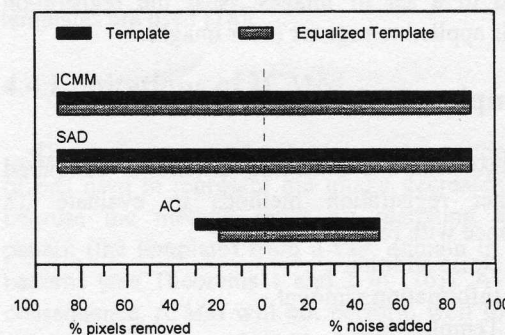


Figure 8: Registration results for the parking image.

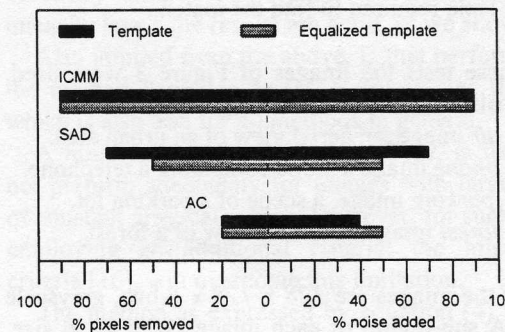


Figure 9: Registration results for the forest image.

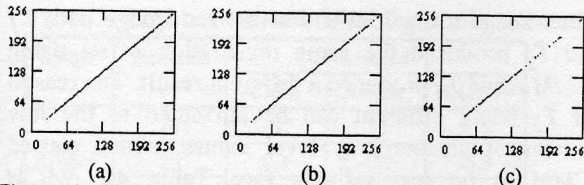


Figure 10: GCP for the *air* image for pixel removal at (a) 10% (b) 50% (c) 90%.

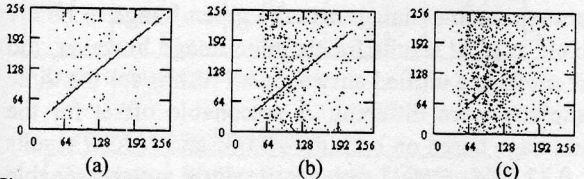


Figure 11: GCP for the *air* image for noise additional at (a) 10% (b) 50% (c) 90%.

Plots of the performance of the three methods with respect to template size are shown in Figures 12 and 13. This was done by taking different template sizes and registering the images. The plots shown are for the maximum range over which the correct registration was found by taking the average results for all the images of Figure 3 as a function of size. Figure 12 is a plot of the registration performance with respect to pixel removal. Figure 13 displays similar result for noise addition. Clearly, the performance of all methods degrade as the size of the template decreases. Once again *ICMM* outperformed both *SADM* and *NACM* for both pixel removal and noise addition.

5.1.3 Time Performance

Figure 14 shows the time performance² of the different methods as a function of the number of search points (*N*) required to perform registration. *N* is calculated by,

$$N = K \cdot L \cdot (M - K + 1) \cdot (N - L + 1) \quad (7)$$

where the image is of dimension *M* x *N*, and the template is of dimension *K* x *L*. We see that *ICMM* has the best performance and *NACM* has the worst. The relationship between time and *N* was found to satisfy a linear relation for all methods. The slopes of the lines of Figure 14, are $2.219(10^{-6})$, $2.978(10^{-6})$ and $7.517(10^{-6})$, for *ICMM*, *SADM*, and *NACM*, respectively. Since the relation is linear, the percent time increase can be calculated directly from the slopes. The *SADM* methods takes 34.2% longer to register images than *ICMM*, and *NACM* takes 238.7% longer to register the images compared to *ICMM*.

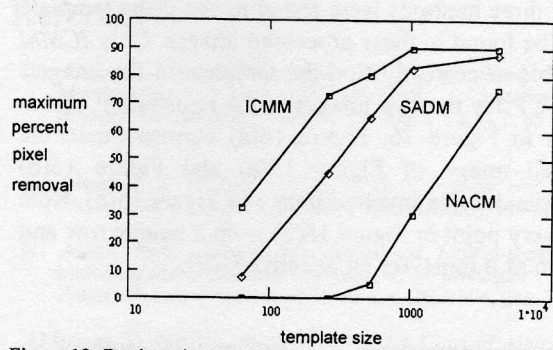


Figure 12: Registration performance for pixel removal versus template size.

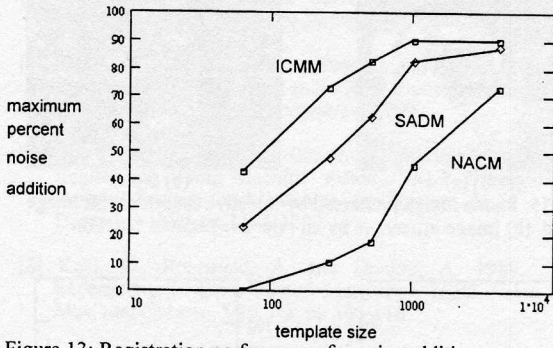


Figure 13: Registration performance for noise addition versus template size.

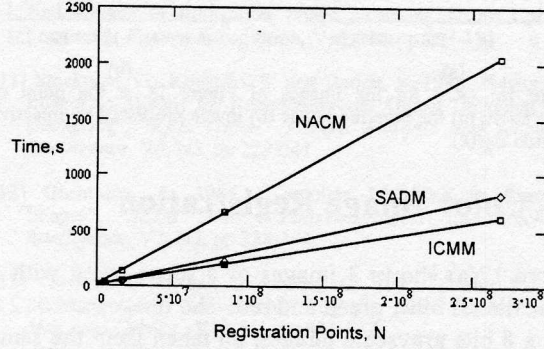


Figure 14: Time Performance of the various methods.

5.1.4 Registering Injective Processed Images

Figure 15 shows the original *Phone* image processed by two other injective functions; $\mathfrak{I}_1(x)$ and $\mathfrak{I}_2(x)$. Figure 15(a) is the phone image inverted by $\mathfrak{I}_1(x) = 255-x$, and Figure 15(b) is the *Phone* image processed by the injective random function,

$$\mathfrak{I}_2(x) = \{\text{random}(x) \mid 0 \leq \mathfrak{I}_2(x) \leq 255, \mathfrak{I}_2(x_1) \neq \mathfrak{I}_2(x_2)\}$$

² Performance measured on a i486 PC - 33 MHZ worksatation.

All three methods were tested to see if the template could be found in these processed images. Only *ICMM* was able to correctly find the template in the images. The *GCP* for the two images at the registration plot is shown in Figure 16. Figure 16(a) correspond to the inverted image of Figure 15(a) and Figure 16(b) correspond to the inverted image of Figure 15(b). Note that every point in Figure 16(b) is on a unique row and column as it must, for an injective *GCP*.

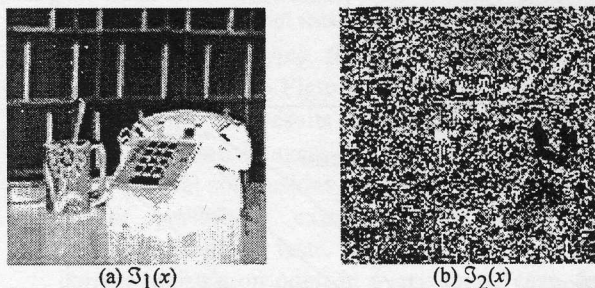


Figure 15: Phone image processed by injective functions. (a) image inverted. (b) Image processed by an injective random function.

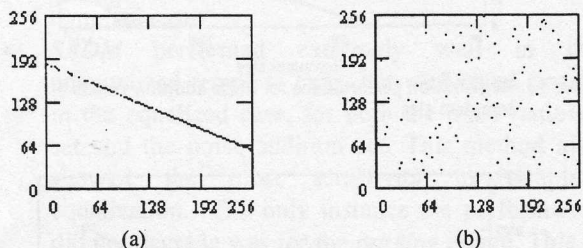


Figure 16: *GCP* for the images of Figure 15 at the point of registration; (a) the inverted image (b) image processed by injective function $S_2(x)$.

5.2 Color Image Registration

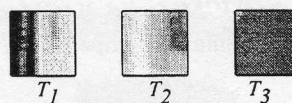
Figure 17(a) shows 3 images of a scene taken with 3 color filters; blue, green and red. The images are $512 \times 256 \times 8$ bits grayscale images, all taken from the same location under the same lighting conditions. They are slightly mis-aligned due to small translational offsets as a result of mounting the filters onto the camera lens. Since there is no way to accurately determine the correct registration points, 3 randomly chosen sub regions of the blue image are taken as registration templates (see Figure 17(b)), and the results of registering these templates with the images will determine the registration offsets. Since the images are known to be misaligned by only a few pixels, the image registration domain used was ± 5 pixels from the location of the template in the blue image.

Table 3 shows the registration results using *ICMM*, *SADM* and *NACM*. The results show that all three templates produced the same registration offset for the

green image using *ICMM*. For the red image both T_1 and T_2 produced the same registration offset using *ICMM* while T_3 produced a different result. The reason for T_3 being different can be attributed to the low number of distinct gray level values, which caused *ICMM* to be less reliable (see Table 4). *NACM* produced 2 results for the green image that are identical to the *ICMM*, which establishes the offset between the blue image and the green image to be 0,1 pixels. *NACM* results for the blue image however, did not produce a unified answer as all results for the three templates were different. A reasonable offset for the red image based on both the *ICMM* and *NACM* results is 0.33,0.66. *SADM* did not produce any reasonable result for any of the images.



(a)



(b)

Figure 17: (a) The blue, green, and red images of the *Books* scene (b) The 3 templates used for registration. (Turbo C++ is a registered trademark of Borland Int.)

Table 3: Registration Results for the *Books* scene.

Image	Template	Method		
		ICMM	NACM	SADM
Green	T_1	0,1*	0,1	5,-5
	T_2	0,1	-5,2	-5,5
	T_3	0,1	0,1	-
Red	T_1	0,0	1,1	5,-5
	T_2	0,1	-5,4	-5,5
	T_3	4,-4	-2,5	-

* Registration offsets are x,y.

Table 4: Template Properties

Template	Distinct Gray Levels	Entropy
1	122	5.69
2	77	5.79
3	30	3.60

6. Future Work

Further research is being conducted to give *ICMM* increased flexibility by:

- Extending the method to register images that are offset by a rotation or a magnification (scaling).
- Subpixel accuracy.
- Exploring methods of cost reduction.
- Better registration results for color (and multi-spectral) images.

7. Summary

A new registration method, the *Intensity Combinatorial Minimization Method (ICMM)* was presented. *ICMM* performs registration by minimizing the number of grayscale combinatorial pairs between the images being registered. The method draws its strength on several attributes; less noise sensitivity, invariance to injective function processing and efficient calculation.

ICMM registered images correctly when various amounts of noise was present or various amounts of information was missing. It also correctly registered images that were processed by injective functions, such as image equalization and image inversion.

The method when compared to the *Normalized Area Correlation Method (NACM)* and the *Sum of Absolute Difference Method (SADM)* performed as good as these methods or better. The time required for registration was significantly less than those required for *NACM* or *SADM*. *ICMM* was also successful in registering images using smaller templates.

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