ROBUST PARAMETER EXTRACTION TECHNIQUES FOR THE DELTA LOGNORMAL MODEL (ΔΛ)

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ABSTRACT

A number of movement generation models have been proposed over the past few years, among them the Delta Lognormal model. This model which has been proven to be the best fit for the study of simple human movements, has also explained many psychological phenomena that have been observed for several decades. Using this model to analyze data from complex movements like handwriting poses a practical problem to researchers, however, which is the development of robust parameter extraction methods capable of handling the signals of actual movements. In this paper, this problem is examined by describing various approaches to the extraction of parameters with the Delta Lognormal model. Techniques are described for lognormal parameter estimation and are then adapted to the Delta Lognormal model.

RÉSUMÉ

Plusieurs modèles de générations de mouvements ont été proposés depuis quelques années. Parmi ces modèles on retrouve le modèle Delta Lognormal, qui est considéré parmi les meilleurs modèles pour décrire la génération de mouvements simples; et qui permet d'expliquer des lois observées depuis maintenant plusieurs décennies. L'analyse des données proposées par ce type de modèle présente toutefois, une difficulté de taille; soit le développement de méthodes robustes d'extraction de paramètres pour des signaux de mouvements complexes, tels les mouvements générant de l'écriture manuscrite. Dans cet article, on fait le tour de la question en exposant différentes approches pour l'extraction de paramètres appliquées aux courbes à profiles Lognormales. On propose aussi un algorithme général pour l'extraction de paramètres du modèle Delta Lognormal appliqué aux signaux d'écriture manuscrite.

I- INTRODUCTION

Several movement generation models have been proposed over the last few decades [PLA-89a][PLA-93e][ALI-93a][ALI-93b]. Among these, the Delta Lognormal (ΔΛ) model stands out not only because it yields more accurate estimations of velocity profiles for simple movements, but also because it has provided explanations for many phenomena that have been consistently observed over the past decades [PLA-92a][PLA-92b][PLA-93a][PLA-93b][PLA-93c][PLA-93d]. It lends itself to the study of various movements, such as arm, eye, etc.

The movement generation model proposed by Plamondon [PLA-87][PLA-89b][PLA-92b][PLA-93a][PLA-93b] is based on the premiss that a movement is constructed from the superimposition of simple movements. Each simple movement, corresponding to an impulse command, involves two neuromuscular networks: an agonist and an antagonist channel. Each of these channel enables the production of lognormal velocity profiles. The resulting velocity signal follows a delta lognormal profile [PLA-93a][PLA-93b], which is the subtraction of the action of the agonist and antagonist channel.

The analysis of complex movements using the Delta Lognormal model poses however some practical difficulties, due to the fact that delta lognormal equations, as lognormal equations, are nonlinear with respect to most of their parameters. And, in the case of composed movements, such as handwriting, where many simple movements are superimposed, we need to handle typically 3 to 10 delta lognormal equations simultaneously, which means 21 to 70 parameters to extract jointly (7 parameters for each delta lognormal equation).

The development of robust parameter extraction methods for nonlinear equations presents a number of problems. One of the major, is the difficulty of developing approaches that make it possible to ensure the convergence of numerical nonlinear regression methods when there is a large number of parameters. The numerical methods generally used for this type of problem therefore require a judicious choice of initial conditions in order to ensure convergence. This choice becomes even more critical as the number of variables increases, and the development of robust methods capable of guaranteeing convergence in all situations becomes essential for extracting parameters from complex signals, such as velocity signals from handwriting movements, where a significant number of parameters must be estimated jointly.

In this paper, we describe a parameter extraction techniques used for analyzing handwriting signals with the Delta Lognormal model. These techniques have been kept fairly general, however, and can be used for the analysis of any compound signal that can be described by the Delta Lognormal model. We present two parameter estimation approaches here for application to a simple lognormal curve: a numerical method and a graphical method. The graphical approach is then applied to the ΔΛ model, which in turn enables the application of the nonlinear regression techniques during the process of extracting the parameters from the ΔΛ model. Finally, a global approach is described that enables parameter estimation for the superimposed ΔΛ that generally comprise more complex signals such as handwriting signals.

II - ESTIMATING THE PARAMETERS OF A LOGNORMAL CURVE

II.1 - Lognormal: functions and distributions (Λ)[1]

In 1879, Galton [GAL-79] showed that if X_1, X_2,...X_n are random independent and positive variables, then (equation (1)):

(1)

$$T_n = \prod_{j=1}^{n} X_j$$

$$\log(T_n) = \sum_{j=1}^{n} \log(X_j)$$

Thus, if the random variables $\log(X_j)$ are such that the central limit theorem applies, then the distribution of $\log(T_n)$ tends toward a normal distribution when n tends toward infinity.

Distributions of the lognormal type may be defined as distributions of a random variable, the logarithm of which is normally distributed: $N(\mu,\sigma^2)$ [EDW-88] (see equation (2)).

(2)

$$\Lambda(x{:}x_0,\mu,\sigma^2) = \frac{1}{\sqrt{2\pi}\,\sigma(x-x_0)} * e^{-\frac{1}{2\sigma^2}(Ln(x-x_0)-\mu)^2}$$

Where:

$\Lambda(x)$ is definied only for $x>x_0$.

μ: is a scale factor.

σ: is a shape parameter ($\sigma>0$).

and $Ln(x-x_0)$ is normal $N(\mu,\sigma^2)$.

Since then lognormal equations (as normal or gaussian equations) have been used in a multitude of applications. The lognormal equation is nonlinear with respect to its parameters x_0, μ and σ. The equation (1) is also presented in a normalised case where the area under the curve is 1. In a more general situations we use to represent the area under the curve by the letter D.

In the following sections, two methods are presented which enable the estimation of the parameters (D, x_0, μ and σ) for lognormal equations.

II .2 - Estimation of parameters by the method of moments

The first method proposed by Cohen and Whitten [COH-80] may be summarized as follows:

Given the following change of variables: $\beta=\exp(\mu)$, in order to simplify the form of the equation. The form of the lognormal curve then becomes as expressed in equation (3).

(3)

$$\Lambda(x{:}x_0,\beta,\sigma) = \frac{1}{\sqrt{2\pi}\,\sigma(x-x_0)} * e^{-\frac{1}{2\sigma^2}\left(\ln\frac{(x-x_0)}{\beta}\right)}$$

Thus, if a new shape parameter is introduced of the form $\omega=\exp(\sigma^2)$, then the values of the mean E(x), the median Me(x), the mode Mo(x), the variance V(x) and the third moment $\alpha_3(x)$ may be expressed as seen in equations (4). And the mean, variance and third moment may be estimated by equations (5). The following three equations are obtained, with three unknowns x_0, β and ω, equations (6). The third equation (in (6)) may be transformed into equation (7). Thus, x_0, μ and σ may be estimated by x_0^*, μ^* and σ^* (see equations (8)).

(4)

$$E(x) = x_0+\beta\omega^{1/2} \quad \textit{(mean)}$$
$$Me(x) = x_0+\beta \quad \textit{(median)}$$
$$Mo(x) = x_0+\beta\omega^{-1} \quad \textit{(mode)}$$
$$V(x) = \beta^2(\omega(\omega-1)) \quad \textit{(variance)}$$
$$\alpha_3(x) = (\omega+2)(\omega-1)^{1/2} \quad \textit{(third moment)}$$

(5)

$$E(x) \approx \bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$
$$V(x) \approx s^2 = \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}$$
$$\alpha_3(x) \approx a_3 = \frac{\sum_{i=1}^{n}(x_i-\bar{x})^3}{n} \Big/ \left(\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}\right)^{3/2}$$

(6)

$$x_0+\beta\omega^{1/2} = \bar{x}$$
$$\beta^2(\omega(\omega-1)) = s^2$$
$$(\omega+2)(\omega-1)^{1/2} = a_3$$

(7)

$$a_3^2=(\omega+2)^2(\omega-1)=\omega^3+3\omega^2-4$$
$$\omega = \sqrt[3]{1+\frac{a_3}{2}\left(a_3+\sqrt{a_3^2+4}\right)} + \sqrt[3]{1+\frac{a_3}{2}\left(a_3-\sqrt{a_3^2+4}\right)} - 1$$

(8)

$$\omega^*=f(a_3) \rightarrow \sigma^*=\sqrt{\ln(\omega^*)}$$
$$\beta^*=\frac{s}{\sqrt{\omega^*(\omega^*-1)}} \rightarrow \mu^*=\ln(\beta^*)$$
$$x_0^*=\bar{x}-\beta^*\sqrt{\omega^*}$$

In the case of distribution analysis the area under the curve should be equal to 1. In the case of a more general equation the area D should be estimated by a numerical integration of the function profile.

This method involves fairly intensive calculations and has some obvious weaknesses. First, these calculations consist in manipulating differences raised to powers of two (estimation of variance) and three (estimation of the third moment), and are therefore highly sensitive to signal noise and involve non negligible errors on the estimation of μ, σ and x_0. The authors [COH-85] were fully aware of this weakness, and as a result they proposed a modified method that obviated the need to estimate α_3. The second defect in this method (but not the least important one) stems from the fact that there is no analytical or heuristic means to evaluate the quality of the parameters estimated. In other words, it can never be known whether or not the distribution or the equation that one is trying to estimate using a lognormal is in fact suited to this type of estimation, and, if it is, to what extent. Because of these defects, another approach came in use which, in spite of its simplicity, enabled the total or partial solution of these problems.

II.3 - Graphical parameter estimation

The graphical approach described in this paper appears in [WIS-66] and enables the more general testing of whether or not the data seem to coincide with lognormal profiles, and, if so, makes it possible to estimate their parameters.

It should be noted, however, that the method described in this section is purely heuristic and has no rigorous analytical basis.

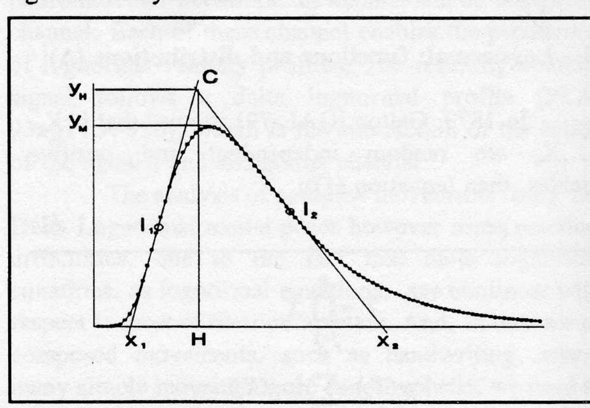

Figure 1 Lognormal Λ.

The graphical method works as follows: two inflection points I_1 and I_2 are located on the curve (see figure 1). Slopes B_1 and B_2 can then be measured from the two tangents to the curve that pass through these inflection points. σ is estimated solely from the ratio of these slopes, where B_1 is the ascending and B_2 the descending (see equation (9)).

(9)

$$\log(B_1/B_2) = 2\sinh^{-1}(\frac{1}{2}\sigma) + \sigma\sqrt{1+\frac{1}{4}\sigma^2}$$
$$= 2\sigma + \frac{1}{12}\sigma^3 - \frac{1}{320}\sigma^5 + \frac{19}{14336}\sigma^7 - .....$$

This function in σ may be estimated by inverse linear interpolation from the data presented in Table I, which covers the values of σ between 0.01 and 0.80. The lower limit x_0 is estimated solely from the points of intersection of the tangents with the X axis (x_1 and x_2 (see equations (10)).

(10)

$$x_0 = \frac{1}{2}(x_2+x_1) - \frac{1}{2}\frac{(x_2-x_1)}{L}$$

$$L = \left[1+\phi Tanh\left(\sigma\sqrt{1+\frac{1}{4}\sigma^2}\right)\right]\left[Tanh\left(\sigma\sqrt{1+\frac{1}{4}\sigma^2}\right)+\phi\right]^{-1}$$

$$\phi = \frac{1+\frac{1}{2}\sigma^2}{\sigma\sqrt{1+\frac{1}{4}\sigma^2}}$$

The divisor L can also be estimated by linear interpolation from the values presented in Table I. Finally, μ is estimated by the following function (11).

(11)

$$\mu = \frac{3}{2}\sigma^2 + \frac{1}{2}\log((x_1-x_0)(x_2-x_0))$$

Once these parameters have been estimated, the quality of the estimations must be evaluated. This is done by comparing them with the ratio y_H/y_M. A good approximation of this ratio is given by the equation (12), which gives a quick way of evaluating the quality of the estimates.

(12)

$$\frac{y_H}{y_M} = 1.213 - 0.05\sigma^2$$

The last estimation that can be extracted from these measures is the area (D) under the lognormal curve. By measuring the area (A) of the triangle (C,x_1,x_2), area D can be estimated using the following relation (13).

(13)

$$\frac{D}{A} = 1.0332\ (1+\frac{1}{3}\sigma^2)$$

This very simple approach yields very good estimations of the parameters σ, μ, x_0 and D. Also, this method offers a simple heuristic for responding to a fundamental question: To what extent is the data that we are trying to model related to a Λ profile?

The quality of the estimations obtained might be improved by using the method of moments or the graphical method, and by applying more accurate numerical methods such as the Levenberg-Marquardt nonlinear regression techniques [MAR-63], the initial conditions for which being the parameters estimated by one of these two methods. This would mean that the divergence effect of the numerical methods would be avoided and their convergence time minimized.

III - ESTIMATION OF THE PARAMETERS OF A SUCCESSION OF ΔΛ CURVES

The $\Delta\Lambda$ curves, with respect to the model proposed by Plamondon [PLA-93a] represents the impulse response of the neuromuscular systems involved with the production of a simple movement. The resulting velocity profiles are thus the subtraction of two synchronous Λ, starting at the same moment t_0 representing respectively the agonist and antagonist activity of the system. The general equation of the $\Delta\Lambda$ model for a simple velocity profile is expressed by equation (14).

(14)

$$\Delta\Lambda(t) = \frac{D_1}{\sqrt{2\pi}\sigma_1(t-t_0)} * e^{-\frac{1}{2\sigma_1^2}(\ln(t-t_0)-\mu_1)^2} - \frac{D_2}{\sqrt{2\pi}\sigma_2(t-t_0)} * e^{-\frac{1}{2\sigma_2^2}(\ln(t-t_0)-\mu_2)^2}$$

The analysis of complex signals, such as velocity signals for the generation of handwriting, generally involve a succession of $\Delta\Lambda$ superimposed (see figure 2). In extracting the parameters of these $\Delta\Lambda$

equations, we must therefore consider the processed signals as superimposition of ΔΛ. Because many of the signals are superimposed in addition to being sequenced in time, however, these curves must be processed jointly and simultaneously.

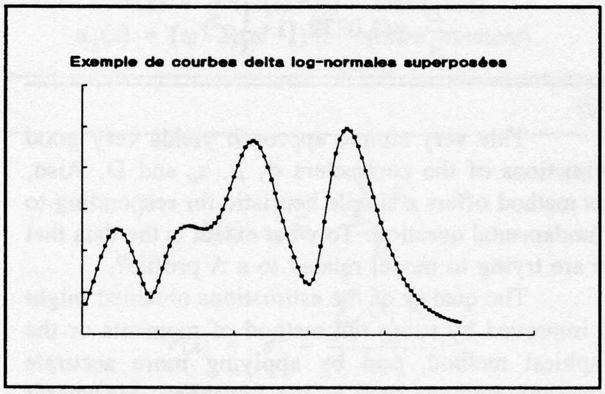

Figure 2 Example of superimposed ΔΛ curves

Parameter estimation can therefore only be carried out iteratively: the parameters of a ΔΛ curve are first estimated roughly and then iteratively on the signal extract to eliminate the effect of its superimposition on the rest of the signal. This step is repeated until the deviation between the old and new estimates is smaller than a certain threshold. In practice, two iterations often prove to be sufficient. The algorithm installed for this step of the processing is as we can see beside (algorithm #1). Figure 3 shows the estimation of the parameters of the second ΔΛ curve (L2). The first iteration yields a rough estimation of the parameters of each ΔΛ. During the second iteration, the parameters of a particular ΔΛ are evaluated with greater accuracy, since the adjacency effec. is subtracted from the other ΔΛ.

```
- Estimation of the ΔΛ
- parameters.

  Initialize all of the N ΔΛ
  parameters at 0.

  * REPEAT the process X times

    Initialize: Signal =
    Original - Estimations

        Add the estimation of
        the nth ΔΛ.

        Estimate the nth ΔΛ in
        the region [a,b]n

        Substract the new esti-
        mation from the nth ΔΛ

      * REPEAT for each ΔΛ
```

Algorithm #1

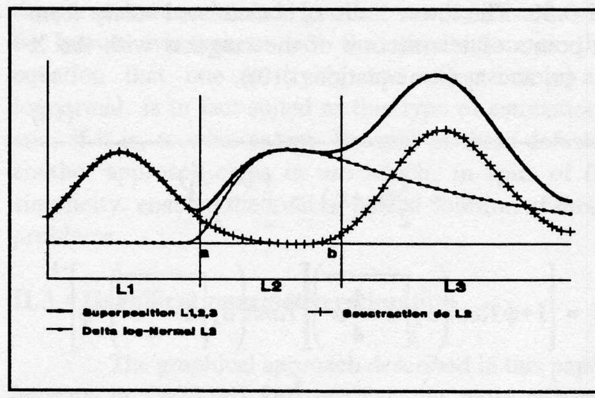

Figure 3 Parameters estimation

The parameters of a ΔΛ curve are estimated in two passes. The first uses the graphical method described previously which enables estimation of the parameters of a Λ curve; these parameters are used in the ΔΛ model simply by dividing the area D between the positive Λ and the negative Λ in such a way that $D_1 > D_2$ and $D = D_1 - D_2$. The second pass, which makes greater accuracy possible by leaving all the parameters free ($\sigma_1 \neq \sigma_2$ and $\mu_1 \neq \mu_2$), is in fact the Levenberg-Marquardt nonlinear regression method [MAR-63]. This method is applied for each ΔΛ curve taken individually in the signal. (The entire method is repeated for the desired number of iterations. Two iterations are generally sufficient.)

It may happen that parameter estimation of the ΔΛ curve cannot be performed, particularly during the first iteration, because of extensive superimpositions involving two adjacent ΔΛs. In this case, estimation of the ΔΛ curve is set aside while the next curves are estimated. Estimation of the preceding ΔΛ curve becomes possible once its immediate neighbours to the left and right have been estimated.

Once a good estimation of the parameters of each of the ΔΛ curves has been obtained, a global evaluation of all the parameters is carried out simultaneously, using the Levenberg-Marquardt nonlinear regression method [MAR-63].

IV- RESULTS

The extraction techniques described before were used for analyzing handwriting samples (figure 4), where the curvilinear velocity profiles were estimated with the superimposition of $\Delta\Lambda$ curves.

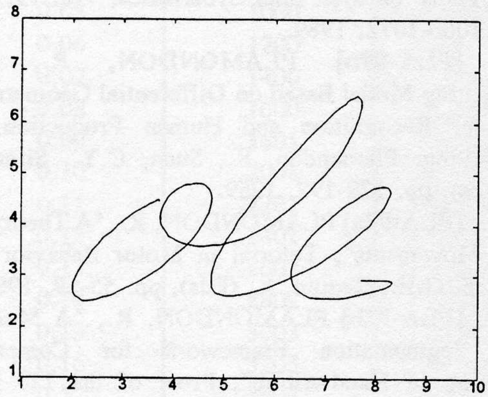

Figure 4 Handwriting sample

In this particular case we have to estimate the parameters of 9 $\Delta\Lambda$ curves superimposed which means 63 parameters simultaneously. The first estimation of the parameters was realised with the graphical method, as shown by the dotted curve in figure 5, combined with one iteration of the nonlinear regression process. All the process can take around 10 seconds on an IBM compatible machine (486Dx at 33MHz). On this simple example the method of the moment has failed in finding an approximation to the Λ curves. As we can see in figure 5, the result of the graphical method are good enough to be considered as an acceptable solution. The mean square error between the original and the reconstructed signal for this example is around 4.24 cm^2/s after 1 iteration of the nonlinear regression process.

The global approach is then used as an optimising process. The result of parameters extraction can be seen in figure 6 where we present the superimposition of the original curve and the estimate (dotted curve) after a 150 iterations of the nonlinear regression process (each iteration can take over 8 sec). The optimising process has permitted to reduce the mean square error to around 1.29 cm^2/s, which is the best we can do in this kind of situation in "reasonable" processing time (around 20 minutes for this example).

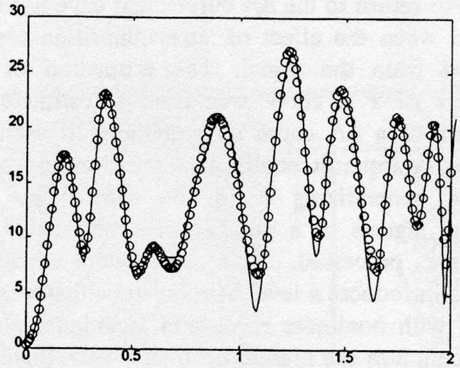

Figure 5 Curvilinear velocity estimated with the graphical method

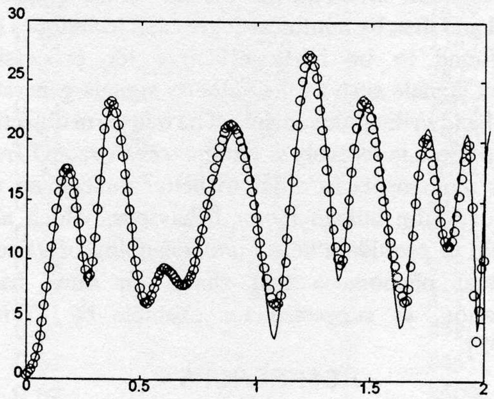

Figure 6 Curvilinear velocity estimated with the global approach

V- CONCLUSION

The parameter estimation methods presented in this paper for solving the problem of extracting the parameters of a lognormal curve (Λ) are a graphical approach and an analytical approach. However in practice through tests on actual cases, we have found that the analytical method is highly sensitive to signal noise. This sensitivity is the result of manipulating differences that are often raised to powers of two or three. When computing the moments, the effect is that the importance of noise is amplified relative to that of the signal. This technique was rapidly abandoned, therefore, in favor of the graphical approach, which, in spite of its simplicity, offers very satisfying estimations of the parameters of a lognormal curve, in a calculation time that is very reasonable. In addition, this approach makes it possible to determine quickly whether or not part of a signal can be modeled by a $\Delta\Lambda$ curve, and, if not, the decision can be made to move to the next $\Delta\Lambda$ curve until estimation of the previous curve is possible, and then to return to the ΔΛ curves that have not been estimated when the effect of superimposition can be subtracted from the signal. The estimation of the parameters of a Λ curve was used to estimate the parameters of a ΔΛ curve by combining it with the Levenberg-Marquardt nonlinear regression approach, and then generalizing it to the case where the compound signals of a number of concatenated ΔΛ curves are processed. This parameter extraction technique introduces a level of robustness that was not achieved with nonlinear regression techniques alone. These techniques can present problems when there is a large number of parameters, since they will only converge with a judicious choice of initial conditions. The estimation of initial parameters by the graphical method and then by nonlinear regression techniques has been found to be fairly effective for processing complex signals such as the velocity signals generated during handwriting movements. The mid-term objective of the project is to analyze parameters extracted from various movements in order to better understand the laws governing human motor behaviour, which also will help to provide a better understanding of various perceptual phenomena that share the same basic information, as suggested for example by Viviani [VIV-92].

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TABLE I

σ	$\log(B_1/B_2)$	L	σ	$\log(B_1/B_2)$	L
0.01	.0200	.0200	0.41	.8257	.6782
0.02	.0400	.0400	0.42	.8461	.6890
0.03	.0600	.0600	0.43	.8666	.6996
0.04	.0800	.0799	0.44	.8870	.7099
0.05	.1000	.0997	0.45	.9075	.7199
0.06	.1200	.1194	0.46	.9280	.7297
0.07	.1400	.1391	0.47	.9486	.7391
0.08	.1600	.1587	0.48	.9691	.7483
0.09	.1801	.1781	0.49	.9897	.7572
0.10	.2001	.1975	0.50	1.0103	.7659
0.11	.2201	.2166	0.51	1.0310	.7743
0.12	.2401	.2356	0.52	1.0516	.7822
0.13	.2602	.2545	0.53	1.0723	.7903
0.14	.2802	.2731	0.54	1.0930	.7980
0.15	.3003	.2916	0.55	1.1137	.8054
0.16	.3203	.3098	0.56	1.1345	.8125
0.17	.3404	.3278	0.57	1.1553	.8195
0.18	.3605	.3456	0.58	1.1761	.8262
0.19	.3806	.3632	0.59	1.1969	.8327
0.20	.4007	.3805	0.60	1.2178	.8390
0.21	.4208	.3976	0.61	1.2387	.8451
0.22	.4409	.4144	0.62	1.2596	.8509
0.23	.4610	.4309	0.63	1.2805	.8566
0.24	.4811	.4472	0.64	1.3015	.8621
0.25	.5013	.4631	0.65	1.3226	.8674
0.26	.5214	.4788	0.66	1.3436	.8725
0.27	.5416	.4942	0.67	1.3647	.8775
0.28	.5618	.5093	0.68	1.3858	.8822
0.29	.5820	.5241	0.69	1.4069	.8868
0.30	.6022	.5386	0.70	1.4281	.8913
0.31	.6225	.5528	0.71	1.4493	.8956
0.32	.6427	.5668	0.72	1.4706	.8997
0.33	.6630	.5803	0.73	1.4918	.9037
0.34	.6833	.5936	0.74	1.5131	.9075
0.35	.7036	.6066	0.75	1.5344	.9112
0.36	.7239	.6193	0.76	1.5558	.9147
0.37	.7442	.6317	0.77	1.5772	.9182
0.38	.7645	.6437	0.78	1.5987	.9215
0.39	.7849	.6555	0.79	1.6202	.9246
0.40	.8053	.6670	0.80	1.6417	.9277

1. "Life is . . . multiplicative rather than additive; the log-normal distribution is more normal than the normal", Anon in "The Scientist Speculates", page 213 (ed. I.J. Good, Heinemann, 1962).