An Introduction to Multi-robot Simultaneous Localization and Mapping

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Overview

A tutorial on multi-robot **S**imultaneous **L**ocalization **A**nd **M**apping

The objective of this presentation is to:

- Introduce the multi-robot SLAM problem
- Mathematically formulate the multi-robot SLAM problem
- Examine the difficulties in performing multi-robot SLAM
- Present various solutions to the multi-robot SLAM problem
- Briefly discuss the open problems

This tutorial assumes that the audience

- has some experience in state estimate
- understands the general concept of SLAM for a single robot

Multi-robot SLAM Overview

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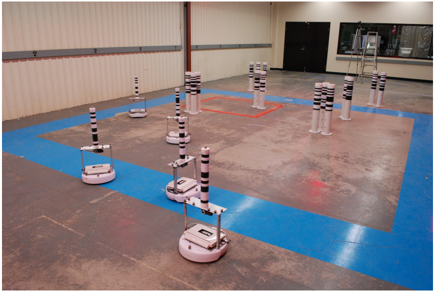
Robots make relative measurements to other robots and landmarks
Each robot must estimate the state of all robots and landmarks

Motivation

Why do we want to use multiple robots?



- Redundancy prevents a single robot causing system failures (if the system is decentralized)
- Potentially achieve a greater reduction in uncertainty for robot pose and landmark position estimates
- Improve data association (if robots can be uniquely identified)
- Distribution of mapping task

Single-robot SLAM

System Model

$$\mathbf{x}_{r,k} = \mathbf{g}(\mathbf{x}_{r,k-1}, \mathbf{u}_k, \epsilon_k) \tag{1}$$

$$\mathbf{x}_{m,k} = \mathbf{x}_{m,k-1}, (\forall m \in M) \tag{2}$$

$$\mathbf{y}_{r,k}^{m,r} = \mathbf{h}(\mathbf{x}_{r,k}, \mathbf{x}_{m,k}, \delta_k), (\forall m \in M_k) \tag{3}$$

Nomenclature:

$\mathbf{x}_{r,k}$	robot state at timestep k
$\mathbf{g}(\cdot)$	robot motion model
$\mathbf{u}_k$	odometry measurement at timestep k
ϵ_k	process noise
$\mathbf{x}_{m,k}$	landmark m state at timestep k
M	the set of all known landmarks up to timestep k
$\mathbf{y}_{r,k}^{m,r}$	relative measurement of landmark m in the robot's body frame
$\mathbf{h}(\cdot)$	measurement model
δ_k	measurement noise

Single-robot SLAM

Let

$$\mathbf{x}_k = \begin{bmatrix} \mathbf{x}_{r,k} \\ \mathbf{x}_{m,k} \end{bmatrix}, \forall m \in M_k \tag{4}$$

We want the estimate

$$\text{bel}(\mathbf{x}_k) := p(\mathbf{x}_k | \text{bel}(\mathbf{x}_0), \mathbf{u}_{1:k}, \mathbf{y}_{r,1:k}^{m,r}) \tag{5}$$

where:

$p(\cdot)$	is a probability density function
$\text{bel}(\mathbf{x}_0)$	is the initial estimate

Note:

- A number of Bayesian filtering methods can be used to find $\text{bel}(\mathbf{x}_k)$ (i.e., the EKF, UKF, PF)
- Data association is required to determine the correspondence between measurements and landmarks

Multi-robot SLAM

System Model

$$\mathbf{x}_{r,k} = \mathbf{g}(\mathbf{x}_{r,k-1}, \mathbf{u}_{r,k}, \epsilon_k), (\forall r \in N) \quad (6)$$

$$\mathbf{x}_{m,k} = \mathbf{x}_{m,k-1}, (\forall m \in M) \quad (7)$$

$$\mathbf{y}_{r,k}^{m,r} = \mathbf{h}(\mathbf{x}_{r,k}, \mathbf{x}_{m,k}, \delta_k), (\forall r \in N, \forall m \in M_k) \quad (8)$$

Nomenclature:

$\mathbf{x}_{r,k}$	robot state at timestep k
$\mathbf{g}(\cdot)$	robot motion model
$\mathbf{u}_k$	odometry measurement at timestep k
ϵ_k	process noise
N	the set of all robots
$\mathbf{x}_{m,k}$	landmark m state at timestep k
M_k	the set of all known landmarks up to timestep k
$\mathbf{y}_{r,k}^{i,r}$	relative measurement object i in the robot's body frame
$\mathbf{h}(\cdot)$	measurement model
δ_k	measurement noise

Multi-robot SLAM

Let

$$X_k = \{\mathbf{x}_{i,k}\}, (\forall i \in N \cup M) \quad (9)$$
$$Y_{r,k} = \{\mathbf{y}^{i,r}_{r,k}\}, (\forall i \in N \cup M) \quad (10)$$

We want the estimate

$$\text{bel}(X_k) := p(X_k | \text{bel}(X_0), \mathbf{u}_{r,1:k}, Y_{r,1:k}, \forall r \in N) \quad (11)$$

where:

$p(\cdot)$	is a probability density function
$\text{bel}(X_0)$	is the initial estimate

Cooperative Localization

Without landmarks, the cooperative SLAM problem becomes the cooperative localization problem.

System Model

$$\mathbf{x}_{r,k} = \mathbf{g}(\mathbf{x}_{r,k-1}, \mathbf{u}_{r,k}, \epsilon_k), (\forall r \in N) \quad (12)$$

$$\mathbf{y}_{r,k}^{m,r} = \mathbf{h}(\mathbf{x}_{r,k}, \delta_k), (\forall r \in N) \quad (13)$$

Let

$$X_k = \{\mathbf{x}_{i,k}\}, (\forall i \in N) \quad (14)$$

$$Y_{r,k} = \{\mathbf{y}_{r,k}^{i,r}\}, (\forall i \in N) \quad (15)$$

We want the estimate

$$\text{bel}(X_k) := p(X_k | \text{bel}(X_0), \mathbf{u}_{r,1:k}, Y_{r,1:k}, \forall r \in N) \quad (16)$$

Cooperative Localization

Past work in cooperative localization (CL) include:

- Kurazume and Hirose (2000) [1] - introduced the leap-frog method
- Roumeliotis and Bekey (2002) [2] - distributed EKF
- Rekleitis, Dudek, Milios (2002) [3] - tradeoffs between sensing paradigms for CL
- Roumeliotis and Rekleitis (2004) [4] - examined the influence on localization performance by the number of robots
- Trawny and Roumeliotis (2009) [5] - quantized measurements to reduce communication cost
- Leung, Barfoot, Liu (2010) [6] - centralized-equivalent approach in a decentralized and sparsely-communicating system

Considerations for Multi-robot SLAM

In single-robot SLAM, we often ask:

- Do we use a feature-based map or grid-based map?
- How do we perform data association?
- Computational complexity

For multi-robot SLAM, we need address the following:

- Centralized or decentralized approach?
- Constraints on network connectivity
- Cyclic updates
- Out-of-sequence measurements (OOSM)
- Reference frames
- Initial correspondence

Some of the above issues are inter-related. How we deal with the above issues will influence the computational complexity of our solution.

Centralized or Decentralized?

Which robot(s) is performing the state estimation?

- the choice is closely related to constraints on network connectivity

Definitions

- In a *centralized* state estimator, one member processes all the information to obtain an estimate of the system
  - all measurements from all robots are accounted for in determining the state estimate
- In a *decentralized* state estimator, each member determines their own state estimate of the system
  - robots need to know which reference frame to use
- In a *distributed* state estimator, the calculation of the state estimate is spread amongst multiple members

Communication Constraints

Network connectivity is affected by communication range and communication dropouts.

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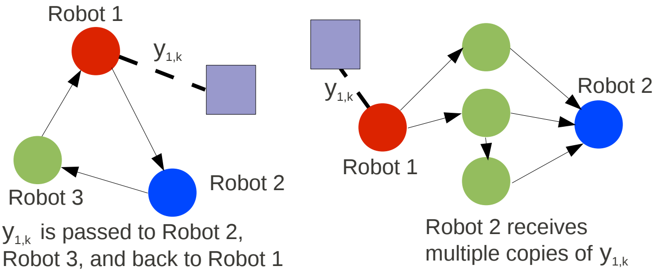
Most work in multi-robot SLAM consider a fully connected network.

Communication bandwidth affects the amount of information exchanged between robots.

Cyclic Updates

Cyclic update is a situation where a measurement is used more than once in updating a state estimate.

- Occurs in decentralized systems
- Produces over-confident (inconsistent) estimates

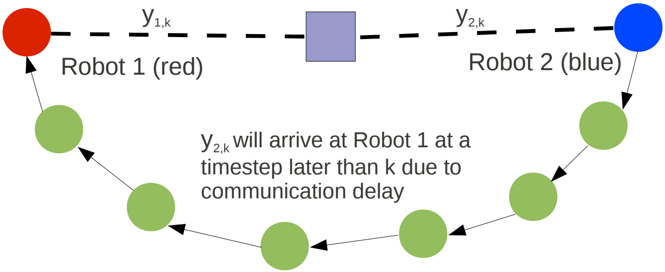


Remedies include:

- *Dependency tree* (not guaranteed to always work) (Howard, A.) [7]
- *Channel filter* (Grime, S.) [8]

Out-of-sequence Measurements (OOSM)

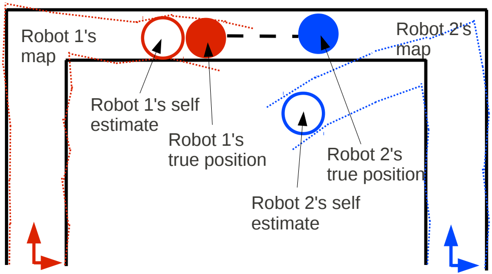
Due to the communication network topology or delays, measurements received by a robot can be out of sequence.



- For a missed measurement, the only way to produce the centralized estimate is to sequentially process all following measurements (Bar-Shalom 2002) [9]

Initial Correspondence

Do robots know of each other's initial pose? Do robots know of their initial relative position?



Relative pose between robots need to be determined or estimated before

- merging maps
- updating estimates with measurements from other robots

Solution to Multi-robot SLAM

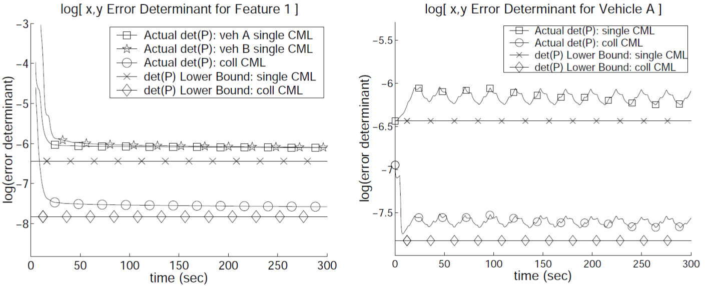
There are many different approaches, many of which are derived from single-robot SLAM solutions.

It is important to keep in mind that besides asking how good the estimate (of robots and map) is, we should also ask how the solution addresses:

- Centralized or decentralized approach?
- Constraints on network connectivity
- Cyclic updates
- Out-of-sequence measurements (OOSM)
- Reference frame selection
- Initial correspondence

Multi-robot EKF SLAM [10]

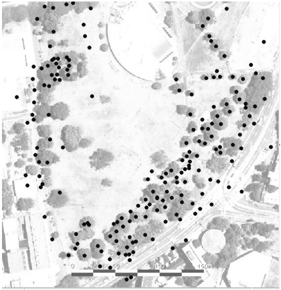
Fenwick, J.W., Newman, P.M., Leonard, J.J. (2002)

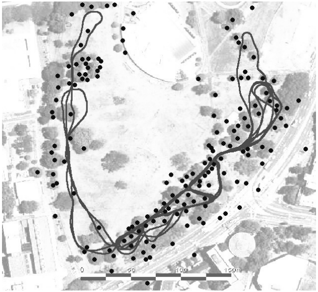


- Generalized EKF-SLAM to the multi-robot case
- Analytically showed how using multiple robots reduces estimate uncertainty
- Centralized, known initial correspondence

Sparse Extended Information Filter [12]

Thrun et al. (2004)





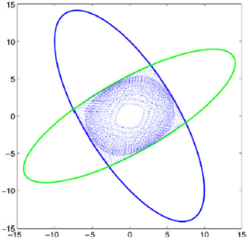
- Sparsification of the information matrix reduces the computational complexity in calculating the state estimate (but this is known to cause estimate inconsistencies [11])
- The filter is generalized for the multi-robot case
- Potentially decentralized, known initial correspondence

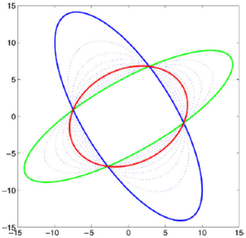
Limited bandwidth decentralized SLAM [14]

Nettleton, E.W., Thrun, S., Durrant-Whyte, H.F., Sukkarieh, S. (2006)

- Communicates submaps which limits communication bandwidth
- The channel filter is used to keep a record of information and prevent cyclic updates
- Covariance intersection algorithm is used to fuse submap information but provides a conservative update

Covariance intersection is a method for combining estimates when the correlations between them are unknown [13]





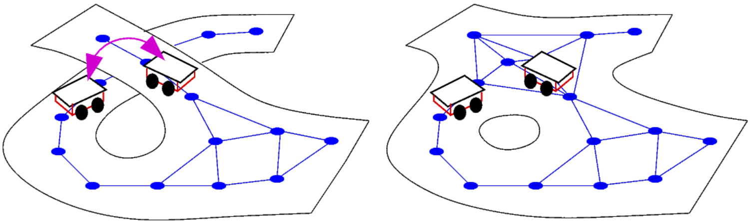
Particle Filter [15]

Howard, A. (2006)

- Uses the Rao-Blackwellized particle filter
- Large dimensionality of state space but very few particles
- Assumes perfect inter-robot observation to determine initial correspondence
- Centralized, unknown initial correspondence

Manifold Maps [16]

Howard, A., Sukhatme, G.S., Matarić, M.J. (2006)



- Manifold of overlapping submaps
- Lazy loop closure
- Robot observations are used for loop closure (joining submaps)
- Centralized, unknown initial correspondence

Manifold Maps [16]

Howard, A., Sukhatme, G.S., Matarić, M.J. (2006)

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- This work has been tested extensively using real robots

Manifold Maps [16]

Howard, A., Sukhatme, G.S., Matarić, M.J. (2006)

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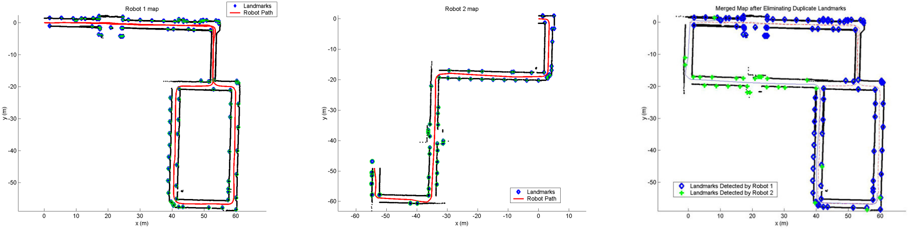
- Another experiment with 4 Robots

Map Merging during Rendezvous [17]

Zhou, X.S., Roumeliotis, S.I. (2006)



- Robots begin mapping independently until rendezvous
- Inter-robot observations are used to align maps from two robots
- Correspondences in the two maps are used to improve alignment
- Centralized, unknown initial correspondence

Map Merging with Planned Rendezvous [18]

Fox, D., Ko, J., Konolige, K., et al. (2006)

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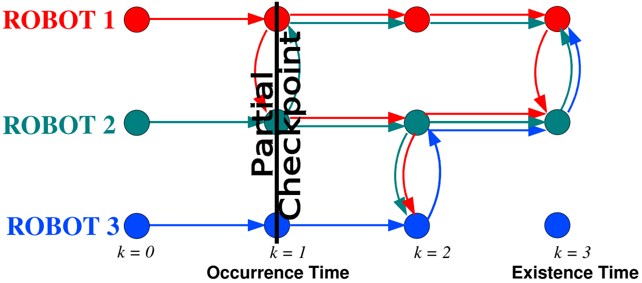
- Map merging occurs after a successful planned rendezvous
- Centralized within a cluster, unknown initial correspondence

The Centralized-Equivalent Approach [19]

Leung, K.Y.K., Barfoot, T.D., Liu, H.H.T. (2010)



- Assumes communication network is never fully connected
- Based on local knowledge, a robot obtains the centralized-equivalent estimate at a *partial checkpoint* and applies the *Markov property*
- Decentralized, known initial correspondence (for now)
- Centralized-equivalent approach avoids cyclic updates

The Centralized-Equivalent Approach

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Open Problems

- not many solutions are decentralized
- not many solutions deal with dynamic networks
- extension from 2d to 3d
- dealing with robot failures

Benchmarking

- the UTIAS ASRL has a 5-robot SLAM dataset available online
- the dataset has groundtruth for all robot poses and landmark positions

Summary

In this tutorial, we covered:

- The basic idea and motivation for using multiple robots for SLAM
- Formulation of the multi-robot SLAM problem, which is similar to the single-robot SLAM problem
- Questions that we need to address in multi-robot SLAM:
  - Centralized or decentralized approach?
  - Constraints on network connectivity
  - Cyclic updates
  - Out-of-sequence measurements (OOSM)
  - Reference frame selection
  - Initial correspondence
- Several existing solutions to the multi-robot SLAM problem
- Open problems

This presentation (pdf) can be found at:
http://asrl.utias.utoronto.ca/∼kykleung/

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