

# Significant Description of 2D Contours by Straight and Curved Segments

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## Abstract

*This paper introduces a new approach for the segmentation and approximation of 2D contours. Its ultimate goal is to find constant curvature segments (straight line segments and/or circular arcs) to describe the contour in a way that respects its actual shape. The approach is strictly based on discrete geometry principles, and the resulting algorithm computes two grouping processes. Preliminary results obtained with this method are compared with two recent methods from the literature. This research work is part of a more generic project for detecting and describing 3D objects in a single 2D image based on high-level structures obtained by perceptual grouping of elementary components.*

## Index terms

*Computer vision, 2D, image curve segmentation, straight line segment, circular arc, shape representation criteria, generic object localization*

## 1. Introduction

The design of robust and effective autonomous systems is one of the main goals of computer vision. An example of such a system is an autonomous mobile robot typically composed of three principal components: (i) a *visual perception* module, (ii) a *reasoning* module and (iii) an *action* module. These components should ideally be organized as a closed-loop information/control data flow allowing the robot to move in its environment and execute application-driven tasks while avoiding collisions.

Under our 3D object detection and description task, the *perception* module of the robot aims at describing its environment based on geometric features of surrounding objects. In a larger context, the feature extraction method developed must be useful both for *generic object recognition* as well as for pose determination of *specific objects*.

The extraction of the initial structural information (basic components such as straight line segments and circular arcs) from a 2D image is critical for the *perception* module since its following processing steps, which consist in the

perceptual grouping of the basic components ([1] [7]) and the volumetric interpretation of these components, depend directly on this information. The volumetric information must lead to a reliable and significant description of the scene [8].

In this paper, we concentrate on the extraction of the basic components using a three-stage process: (i) edge detection, (ii) contour following, and (iii) description of the contour using constant curvature segments which, due to their simple semantics, form an adequate basis for the following grouping process.

Section 2 describes briefly the problem of 2D contour segmentation and approximation. Section 3 compares two existing approaches for the extraction of straight line segments and circular arcs. This is followed by a presentation of a new approach for the segmentation and approximation of 2D contours. This approach rests on several shape representation criteria which lead to more significant descriptions of the contours. The various steps of the three compared algorithms are illustrated and Section 5 discusses their performance on 2D images of medium complexity. Finally, Section 6 concludes and presents future developments.

## 2. Contour segmentation and approximation

The main goal of the segmentation and approximation of 2D contours is to find an optimal combination of straight line and/or curved segments that best describe the shape of the contours. Most man-made objects can be described by straight line segments and, for their more complex parts, by simple curves (circular arcs, elliptic arcs, splines). There exists two principal paradigms for the segmentation of contours: (i) the extraction of significant points joining the primitives (such as the CPS approach [2]), and (ii) the extraction of the primitives forming the contour [4] [5] [10] [11]. The crucial point addressed by these methods is to determine the points along the contour, appropriate to a description, at which it must be broken.

In general, the contours extracted from real 2D images are corrupted by noise. This complicates the unambiguous and accurate extraction of the points of interest. As the approaches belonging to the first paradigm remain sensitive

to noise and are thus discussed less in the literature, the approaches that will be presented in the following sections adopt the second paradigm.

The following section describes two methods from the literature, the approach proposed by Etemadi and the approach proposed by Rosin and West, for the description of contours: the first is based on an edge pixel bottom-up merging and the second on an edge pixel top-down splitting. The new method, that aims at grouping edge pixels into segments of constant curvature which can be approximated by straight line segments and circular arcs, is presented after. This method is also based on the edge pixel bottom-up merging.

### 3. Contour segmentation algorithms

For all contour segmentation algorithms described in this paper, the extraction of edge pixels in 2D images is performed by the Canny edge operator which finds the pixels corresponding to the position of the local maximum of the gradient. Starting at junctions/extremities and ending at junctions or extremities, the open or closed contours forming the outline of the objects are then extracted by following connected pixels.

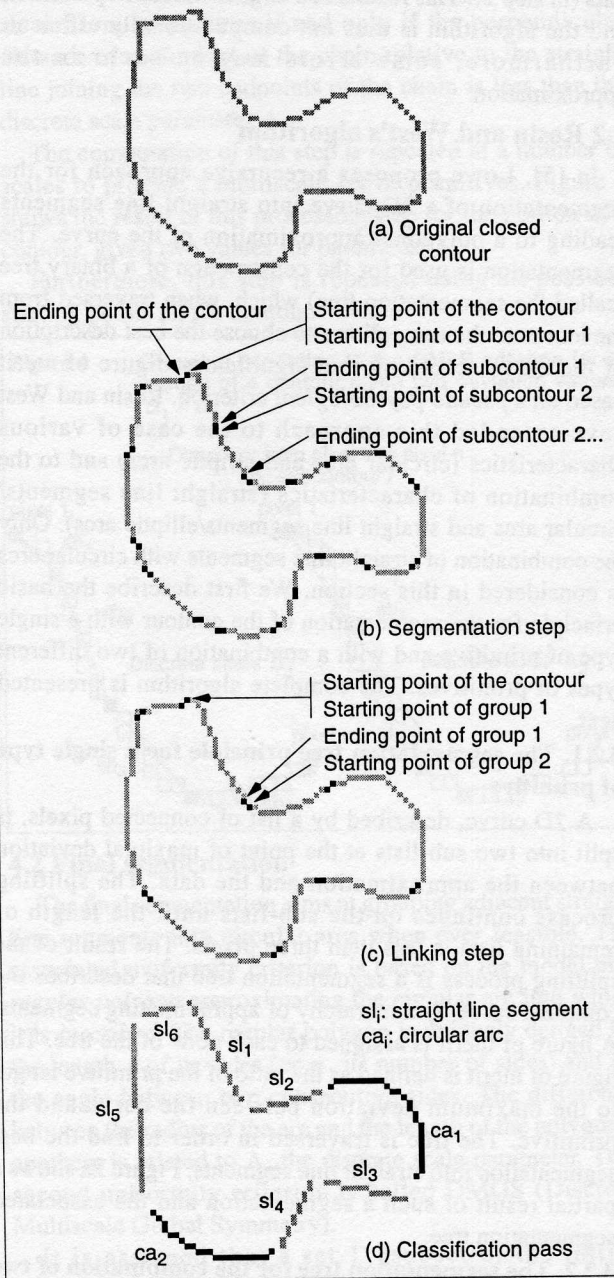
It's interesting to note that, for the three discussed methods, the segmentation process is intrinsically parallel (global process on the whole set of contours) while it is sequentially performed (local process on each contour).

#### 3.1 Etemadi's algorithm

The approach proposed by Etemadi, which is described in detail in [5], allows to segment the 2D image contours into straight line segments and/or circular arcs. The algorithm proceeds according to three processing steps. For each 2D open or closed contour included in the image (Figure 1 shows the result of each step for a synthetic image of a simple closed contour, Figure 1a):

- 1• segment each contour into subcontours under the hypothesis of *local symmetry*, which considers the deviation of a pixel with regard to a straight line segment. It is assumed that a chain composed of three (or less) pixels is symmetrical (each subcontour is bounded by two black pixels in Figure 1b),
- 2• group adjacent subcontours on the basis of co-curvilinearity and co-linearity. The proximity of subcontour endpoints and similar curvature between the subcontours may trigger a grouping (in Figure 1c, each group is bounded by two black pixels),
- 3• classify groups (subcontour or adjacent subcontours) into straight line segments or circular arcs based on the co-linearity principle mentioned at step 2. A given segment is classified as a straight line segment if the maximum deviation is less than two pixels from the central point, otherwise, it is classified as a curved segment (in Figure 1d, straight line segments are

Figure 1 Results obtained with the method proposed by Etemadi on a simple closed contour object. (a) Original closed contour. (b) Segmentation pass. (b) Linking pass. (c) Classification pass (straight line segments shown in grey, curved segments shown in black).



shown in grey while curved segments are shown in black).

Crevier *et al.*, [4], proposed an approach similar to the one by Etemadi. The major difference is in the symmetry test which takes into account explicitly the length (Euclidean length) of the subcontours and which defines an empirical stopping criterion based on the number of pixels of the subcontours.

Etemadi's approach presents several advantages: it is simple, it relies on only a few absolute thresholds, and it is scale invariant. However, the algorithm requires the fitting of a large number of primitives (circular arcs) on the raw data (in step 2). This results in a large number of operations, and the algorithm is thus not computationally efficient. Furthermore, some errors may occur in the approximation.

### 3.2 Rosin and West's algorithm

In [5], Lowe proposes a recursive approach for the segmentation of a 2D curve into straight line segments leading to a polygonal approximation of the curve. The segmentation is used for the construction of a binary tree (called the segmentation tree) which, when traversed from the leaves to the root, allows to choose the best description of the curve according to a significance figure of merit based on a *pseudo-psychological* criterion. Rosin and West have extended this approach to the case of various characteristics (circular arcs and elliptic arcs) and to the combination of characteristics (straight line segments/circular arcs and straight line segments/elliptic arcs). Only the combination of straight line segments with circular arcs is considered in this section. We first describe the basic principle for the segmentation of the contour with a single type of primitive. The complete algorithm is presented next.

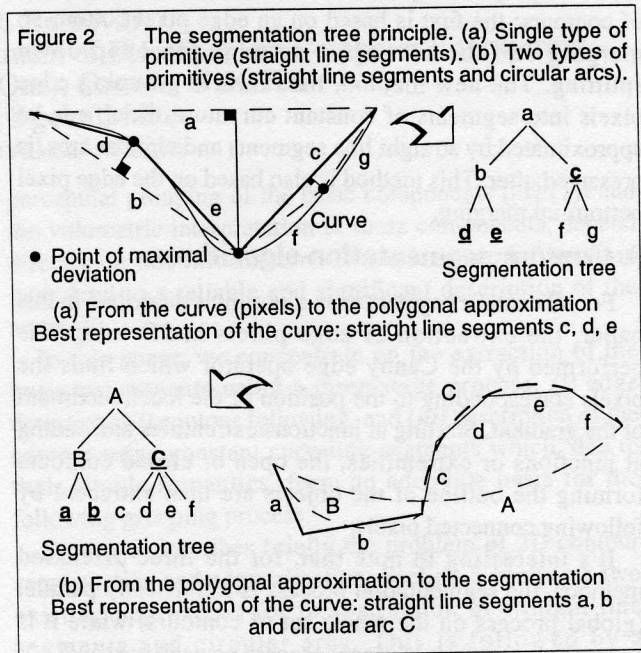
#### 3.2.1. The segmentation tree principle for a single type of primitive

A 2D curve, described by a list of connected pixels, is split into two sub-lists at the point of maximal deviation between the approximation and the data. The splitting process continues on the sub-lists until the length of remaining lists is less than three pixels. The result of the splitting process is a segmentation tree that describes the complete curve by a hierarchy of approximating segments. A figure of merit is assigned to each node of the tree. This figure of merit is defined as the ratio of the primitive length to the maximum deviation between the curve and the primitive. The tree is traversed in order to find the best segmentation into straight line segments. Figure 2a shows a partial result of such a segmentation and the associated segmentation tree.

#### 3.2.2. The segmentation tree for the combination of two types of primitives

When the curve is described by a single type of primitive (e.g. straight line segments), the basic elements belonging to the curve (the individual pixels) are not part of the segmentation tree. Only the primitives (groups of connected pixels) are included in the data structure for the approximation of the curve. However, when the goal is to describe the curve with two different types of primitives,

such as straight line segments and circular arcs, the straight line segments must be included as leaves in the tree structure, since circular arcs are obtained as a combination of straight line segments. Figure 2b shows a partial result of the segmentation of a curve with straight line segments and circular arcs and the associated segmentation tree.



#### 3.2.3. Description of the algorithm

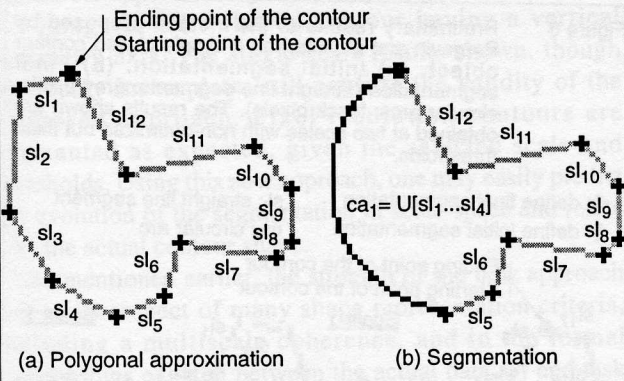
The algorithm proceeds according to the following steps. For each 2D open or closed contour included in the image:

- 1• build a polygonal approximation of the contour (see Figure 2a),
- 2• extend straight line segments by grouping adjacent straight line segments belonging to the same level of the segmentation tree,
- 3• fit circular arcs to sequences of straight line segments,
- 4• extend circular arcs by grouping with adjacent straight line segments and/or adjacent circular arcs belonging to the same level of the segmentation tree.

Figure 3 shows the output of the algorithm following the polygonal approximation step (Figure 3a), and the segmentation step (Figure 3b); straight line segments are shown in grey and curved segments are shown in black. The inaccuracy in the approximation of the curve in the part defined by straight line segments  $sl_1$  and  $sl_2$  is clearly visible. This algorithm also requires a large number of primitive fitting on the computed data and is thus computationally expensive, especially in steps 3 and 4. Furthermore, the recursive splitting process may create false breakpoints on noisy contours.

An older version of the algorithm (see [11]) has been studied by Etemadi in [5] on the basis of two figures of merit, and the conclusions were that his own approach gave

Figure 3 Results obtained with Rosin and West's method on a simple closed contour object. (a) Polygonal approximation. (b) Segmentation in straight line segments (grey dots) and circular arcs (black dots).



a better performance for the detection of circular arcs at multiple scales and was also more appealing from the perceptual standpoint.

#### 4. Overview of the proposed approach

Even though they tackle the problem with significantly different approaches, the two above methods do not always provide a satisfactory approximation of the contours for the following perceptual processes, up to shape recognition. In order to remedy to this problem, Bergevin and Mokhtari [3] proposed a new segmentation and approximation approach based on formal constraints related to a large number of shape representation criteria, including shape preserving, local, multiscale, stable and invariant criteria.

##### 4.1 Basic principle of the proposed approach

Unlike most existing multiscale contour segmentation and approximation methods, the proposed approach is strictly based on discrete geometry principles. As such, it does no linear filtering and no straight line or circular arc fitting.

The resulting algorithm computes, at each scale, a sequence of two grouping processes. Each one is defined as a formal segmentation (in the sense of [9]) and uses a distinct uniformity criterion. The two grouping processes are called: (i) the *initial* segmentation and (ii) the *final* segmentation. The algorithm is parameterized using a small number of thresholds related to the actual data set and task properties. Only contours formed by two or more pixels are considered for processing. The following sections present a more in depth description of the algorithm.

###### 4.1.1. Initial segmentation

The initial segmentation splits each contour into several subcontours, each of which approximated by a straight line segment. The associated point grouping uniformity criterion is equivalent to a discrete *mutual co-circularity* criterion among the discretely connected points of the subcontour. A *discrete scale parameter*  $\Delta$ , acting as a

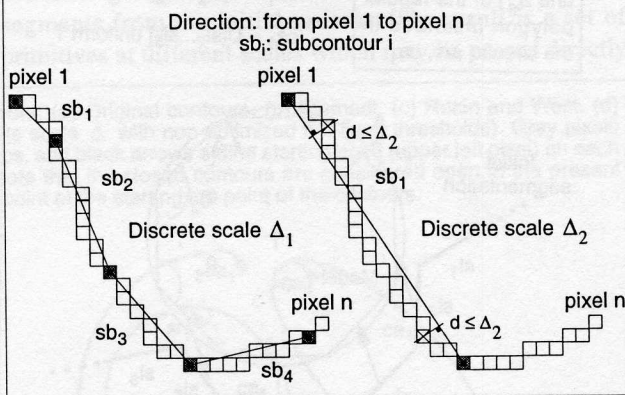
maximum deviation criterion, is associated with a scale measure. This first uniformity criterion is called DMLS (Discrete Multiscale Local Symmetry).

It is assumed that a pixel chain of two or less pixels form an uniform subcontour. A chain of two or more pixels form an uniform subcontour if and only if the perpendicular distance of each pixel of the chain relative to the straight line joining the two endpoints of the chain is less than the discrete scale parameter  $\Delta$ .

The computation of this step is repeated at a number of scales to provide a multiscale set of primitives. Figure 4 shows the result of this processing step for the portion of a contour, given two values for parameter  $\Delta$ .

Furthermore, this step is repeated using all possible starting points on the contour.

Figure 4 Grouping according to the DMLS criterion for a section of a contour given two deviation values  $\Delta_1 < \Delta_2$ .



##### 4.2 Final segmentation

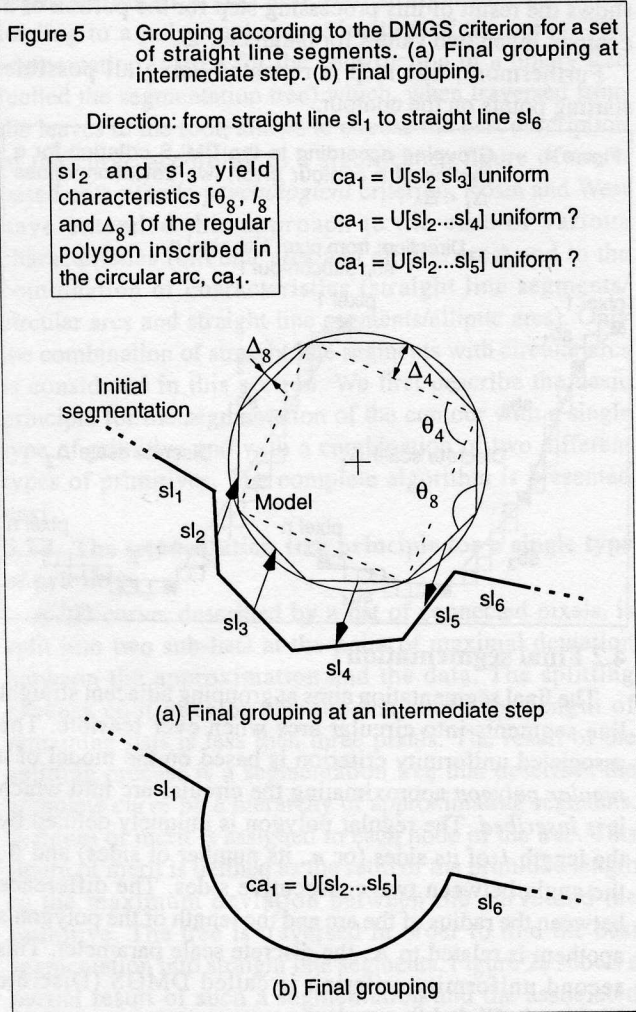
The final segmentation aims at grouping adjacent straight line segments into circular arcs when ever feasible. The associated uniformity criterion is based on the model of a *regular polygon* approximating the circular arc into which it is *inscribed*. The regular polygon is uniquely defined by the length  $l$  of its sides (or  $n$ , its number of sides) and  $\theta$ , the angle between two consecutive sides. The difference between the radius of the arc and the length of the polygon's apothem is related to  $\Delta$ , the discrete scale parameter. This second uniformity criterion is called DMGS (Discrete Multiscale Global Symmetry).

It is assumed that a set  $\Gamma$  (subset of the initial segmentation of a contour) consisting of a single straight line segment is uniform. When a set  $\Gamma$  composed of  $l_\Gamma$  straight line segments is uniform, then the set  $\Gamma'$  composed of  $l_\Gamma = l_\Gamma + 1$  straight line segments is uniform if and only if:

- 1•  $l_\Gamma \geq 2$ ,
- 2•  $l_\Gamma$  is less than  $n$ , the number of sides forming the regular polygon,
- 3• angle  $\theta_{ss'}$  between two consecutive segments  $s \in \Gamma$

- and  $s' \in \Gamma$  is similar to angle  $\theta$  of the regular polygon defined by  $\Gamma$  and inscribed in the circular arc,
- 4• length  $l_{s'}$  is similar to the lengths  $l$  of the sides of the regular polygon,
- 5• the value of  $\Delta_{s'}$ , which is the difference between the radius of the arc and the length of the apothem associated to the polygon defined by  $\theta_{s'}$  and  $l_{s'}$ , is less than  $\Delta$ .

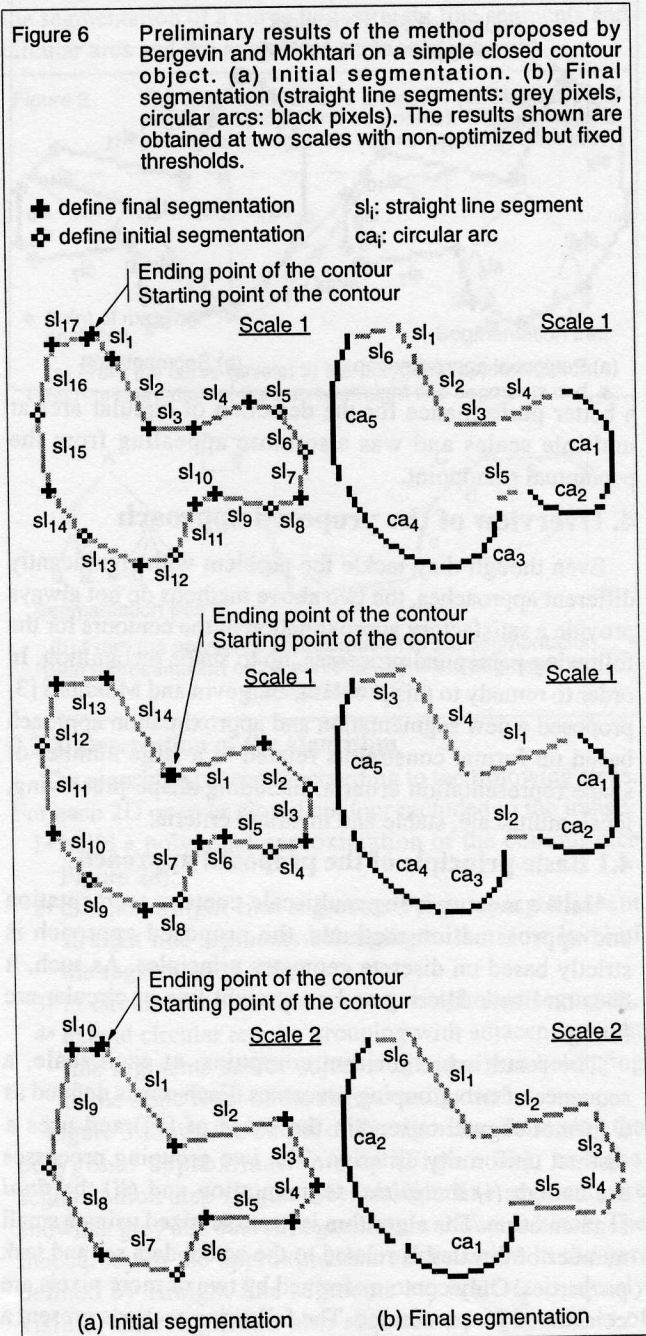
Figure 5 provides an example of the grouping principle based on the DMGS criterion for part of a contour described by the initial segmentation.



Once again, this step is repeated using all possible starting straight line segments from the initial segmentation.

Figure 6 shows preliminary results of the method, given two values for parameter  $\Delta$  ( $\Delta_1 < \Delta_2$ ), for the initial segmentation step (Figure 6a) and the final segmentation step (Figure 6b). Two starting/ending points on the contour are used to start the whole process at scale  $\Delta_1$ . Straight line segments are shown in grey and curved segments are shown in black. These results are shown mainly for illustration purposes, some extracted segments being identical or

simply overlapping. In effect, a multiscale redundant description is first obtained, only a subset of which is to be considered significant with respect to either the actual data set or task properties.



## 5. Comparison of results

Figure 7 and Figure 8 show results obtained using the three methods presented in this paper for simple shape contours (circular arc, circle, ellipse, curve for Figure 7, and circles and ellipses for Figure 8), at multiple scales for the Bergevin and Mokhtari's method (once again, straight line

segments are shown in grey, curved segments are shown in black). The results obtained with the new method (especially on the circles and on the ellipses) justify the multiscale approach. Figure 9 shows additional results for the image of a lis-shaped contour having a vertical symmetry axis. The single-scale results shown, though obviously incomplete, still confirm the validity of the proposed approach. Pixel resolution contours are segmented as expected, given the selected scale and thresholds. Using this new approach, one may easily predict the evolution of the segmentation in scale-space and relate it to the actual contour shape.

As mentioned earlier, the strength of the new approach lies in its respect of many shape representation criteria, including a multiscale coherence, and in the formal relationships existing between the actual data set and task properties, the selected thresholds and the results to be expected (see [3] for more details). A complete experimental validation of the approach is under way.

## 6. Conclusion and future research

First results were presented using a recently proposed contour segmentation and approximation algorithm. This algorithm, strictly based on discrete geometry principles, can be shown to respect a large number of shape representation criteria. It aims at finding the best set of circular arcs and straight line segments to describe the contour shape. The presented results confirm the validity of the approach and its potential to overcome the limitations of previous methods. An interesting aspect of the new algorithm is its significant reduction in computational complexity with respect to previous algorithms, because it does no linear filtering and no straight line or circular arc fitting.

Furthermore, a redundant covering of a contour is obtained, at each scale, by repeating the initial grouping process using all possible starting points on the contour and the final grouping process using all possible starting segments from the initial process. The result is a set of primitives at different scales which may be passed directly

Figure 7 Results of segmentation methods on simple shape contours. (a) Original contours. (b) Etemadi. (c) Rosin and West. (d) Bergevin and Mokhtari (preliminary results, each at a single scale  $\Delta$  with non-optimized but fixed thresholds). Grey pixels define straight line segments, black pixels define circular arcs, and black arrows define starting point (upper left pixel) on each contour. In (d), black crosses define final segmentation, note that the closed contours are considered open in the present implementation which explains the spurious segmentation point at the starting top point of the contours.

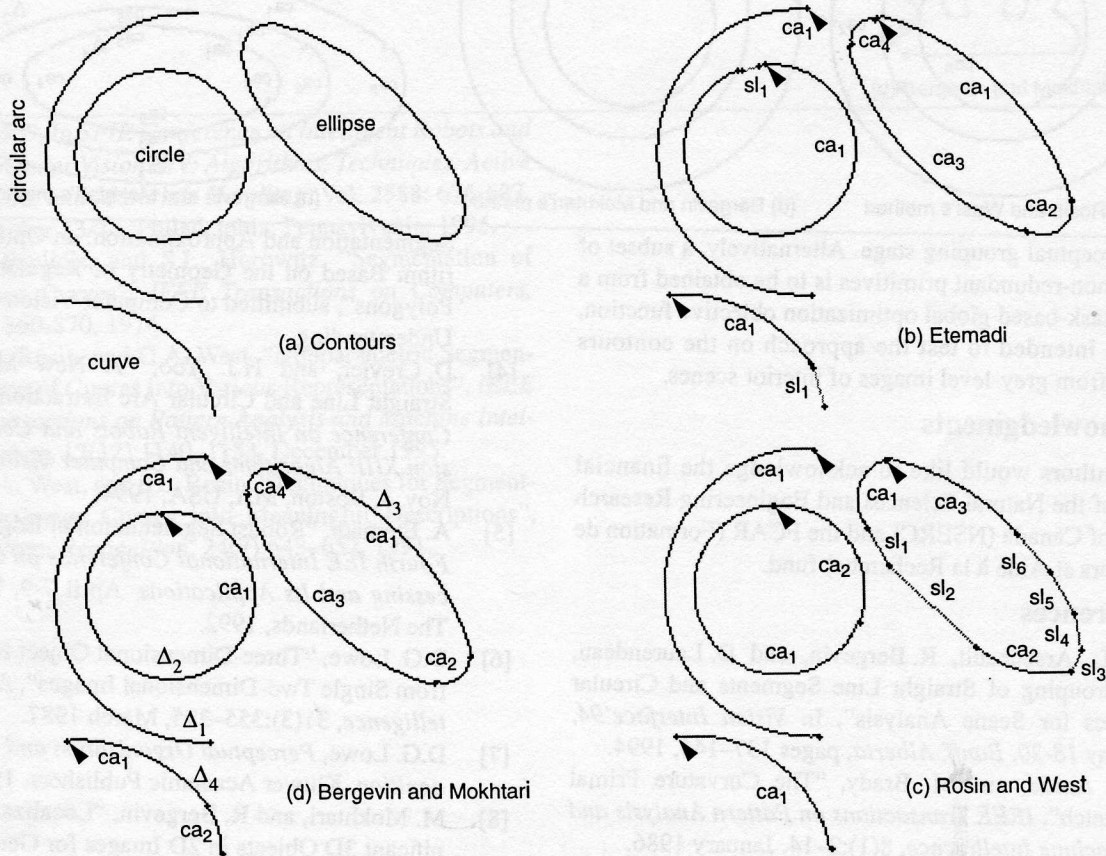
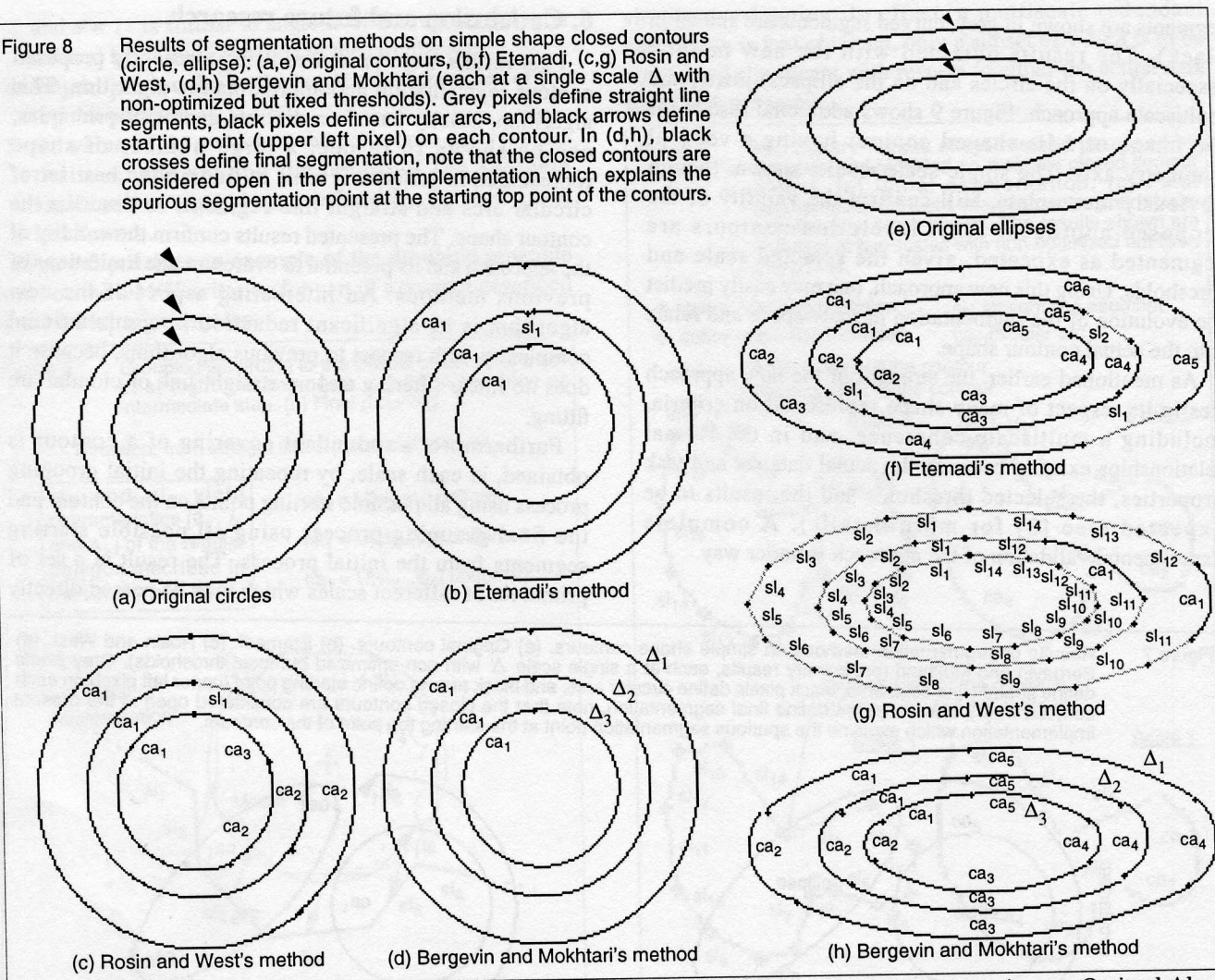


Figure 8

Results of segmentation methods on simple shape closed contours (circle, ellipse): (a,e) original contours, (b,f) Etemadi, (c,g) Rosin and West, (d,h) Bergevin and Mokhtari (each at a single scale  $\Delta$  with non-optimized but fixed thresholds). Grey pixels define straight line segments, black pixels define circular arcs, and black arrows define starting point (upper left pixel) on each contour. In (d,h), black crosses define final segmentation, note that the closed contours are considered open in the present implementation which explains the spurious segmentation point at the starting top point of the contours.



to the perceptual grouping stage. Alternatively, a subset of possibly non-redundant primitives is to be obtained from a possibly task-based global optimization objective function. It is also intended to test the approach on the contours obtained from grey-level images of interior scenes.

## 7. Acknowledgments

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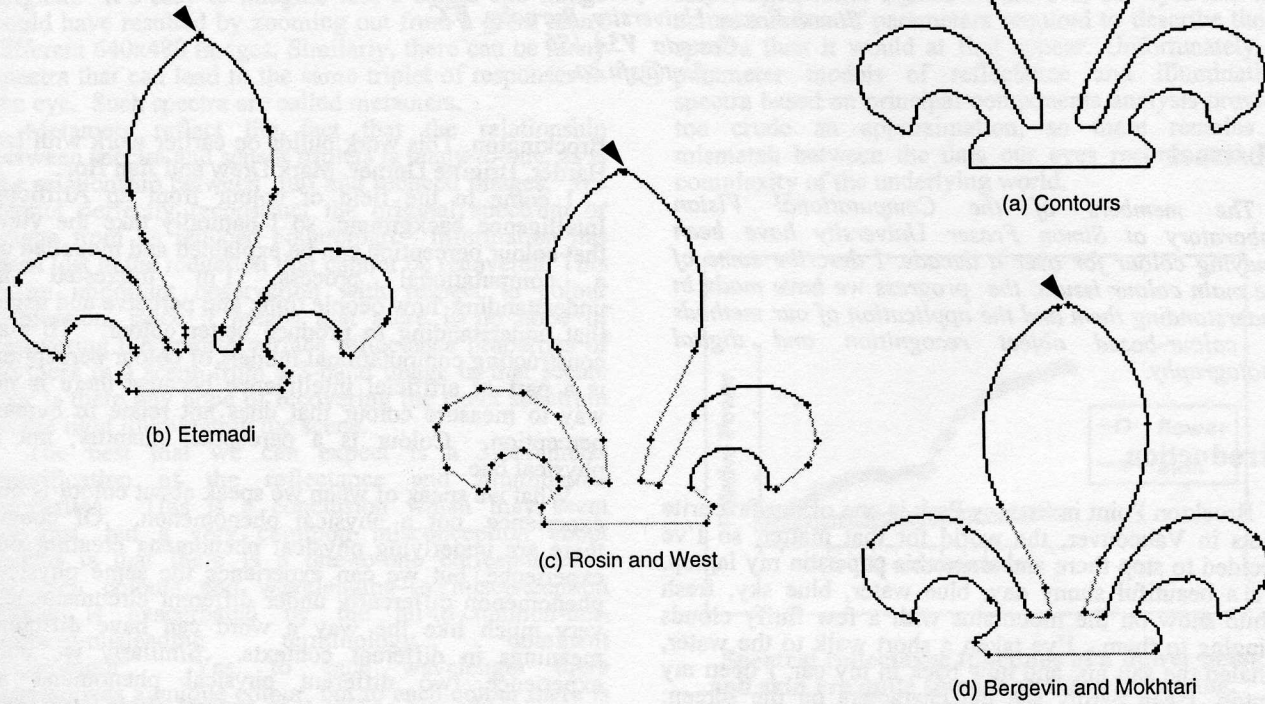
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Figure 9 Results of segmentation methods on a lis-shaped contour. (a) Original contour. (b) Etemadi. (c) Rosin and West. (d) Bergevin and Mokhtari (preliminary results at a single non-optimal scale with non-optimized but fixed thresholds). Grey pixels define straight line segments, black pixels define circular arcs, and black arrows define starting point (upper left pixel) on each contour. In (d), black crosses define final segmentation, note that the closed contours are considered open in the present implementation which explains the spurious segmentation point at the starting top point of the contours.



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