A 3D vision system with only one camera

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Abstract

The 3D vision of an object requires the observation of this object from several different view points. We present herein a method which uses only one camera and a set of mirrors. We show the feasibility of the method and give the relations which allow the calibration of the device as well as the determination of the coodinates, thus the dimensions of the object observed.

1 Introduction

3D reconstruction is in principle not possible with only one 2D view. Classically, either several cameras (stereovision), or one camera associated with a structured lighting system is used. We present herein a 3D vision system with only one camera. The views of an object under different view angles which enable the reconstruction are in fact obtained by a set of mirrors, the object as well its reflections being simultaneously observed by the camera. Firstly, we describe the principle of the system then we will establish the elementary geometrical reflection relations and finally we will indicate the calibration method used.

2 Principle and diagram of the system

2.1 Diagram of the system

The CCD camera is positioned vertically and directed towards the bottom. It allows us to observe the system of height mirrors oriented according to the sides of an upside down octagonal pyramid (figure 1). The object to be reconstructed is placed at the center of the mirrors.

All the sides of the object can thus be viewed except the bottom side. The system is equivalent to a nine-camera system, where eight of them are virtual, reflections of the real camera by the different mirrors.

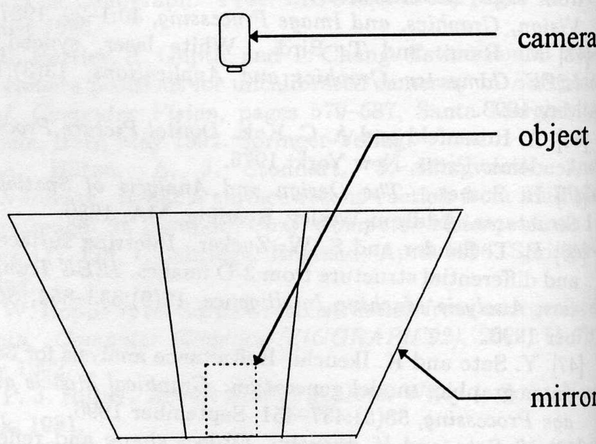

Figure 1: Lateral view of the system.

2.2 Notations

The coordinates of the objects will be given with respect to the reference axes of the camera centered on the center of projection of the latter. The z axis is merged with the optical axis of the camera, and is directed toward the bottom. The coordinates of the points of the image are measured in pixels in a two-dimensional reference associated with the image. We will use the intrinsic parameters of the camera : u_0, v_0, k_u, k_v, f. u_0, v_0 are the coordinates, measured in pixels, of the center of projection or main point in the image reference, k_u, the horizontal scaling factor (in pixels per mm), k_v, the vertical scaling factor and f the focal length. The coordinates in the image reference of $A(u,v)$, the image of a point $A(x,y,z)$ of the object, will be thus be

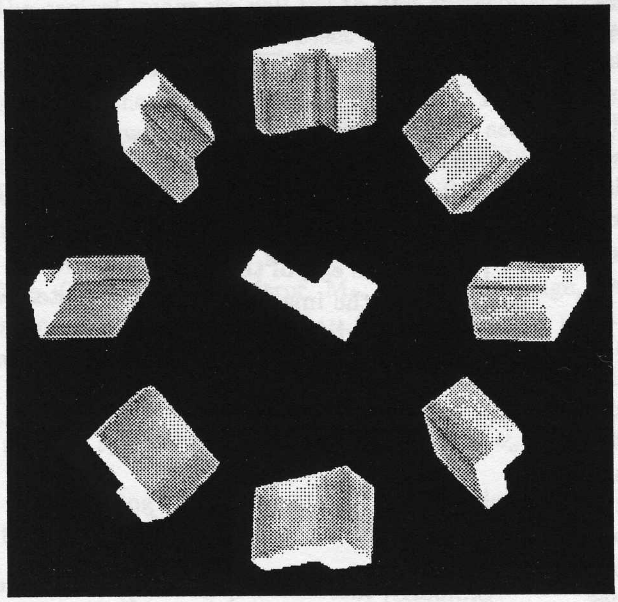

Figure 2: The object and its reflections as seen by the camera.

$$
\begin{aligned}
u &= k_u f \frac{x}{z} + u_0, \\
v &= k_v f \frac{y}{z} + v_0.
\end{aligned} \tag{1}
$$

To simplify, we use the normalized coordinates μ and ν

$$
\begin{aligned}
\mu &= \frac{x}{z}, \\
\nu &= \frac{y}{z},
\end{aligned} \tag{2}
$$

with the relations

$$
\begin{aligned}
u &= k_u f\,\mu + u_0 \Leftrightarrow \mu = \frac{u - u_0}{k_u f}, \\
v &= k_v f\,\nu + v_0 \Leftrightarrow \nu = \frac{v - v_0}{k_v f}.
\end{aligned} \tag{3}
$$

3 Elementary geometrical relations

3.1 Calculating the coordinates of the reflections of a point by a mirror

Let the point A of coordinates (x, y, z). We would like to know the coordinates of $A'(x', y', z')$, the reflection of A by the mirror M of equation $ax + by + cz + d = 0$ (figure 3). We obtain

$$
\begin{aligned}
x' &= \frac{x\left(-a^2+b^2+c^2\right) - 2aby - 2acz - 2ad}{a^2+b^2+c^2}, \\
y' &= \frac{-2abx + y\left(a^2-b^2+c^2\right) - 2bcz - 2bd}{a^2+b^2+c^2}, \\
z' &= \frac{-2acx - 2bcy + z\left(a^2+b^2-c^2\right) - 2cd}{a^2+b^2+c^2}.
\end{aligned} \tag{4}
$$

In the image reference the normalized coordinates of A' would be

$$
\begin{aligned}
\mu' &= \frac{x'}{z'} \\
\mu' &= \frac{x\left(-a^2+b^2+c^2\right) - 2aby - 2acz - 2ad}{-2acx - 2bcy + z\left(a^2+b^2-c^2\right) - 2cd}, \\
\nu' &= \frac{y'}{z'} \\
\nu' &= \frac{-2abx + y\left(a^2-b^2+c^2\right) - 2bcz - 2bd}{-2acx - 2bcy + z\left(a^2+b^2-c^2\right) - 2cd}.
\end{aligned} \tag{5}
$$

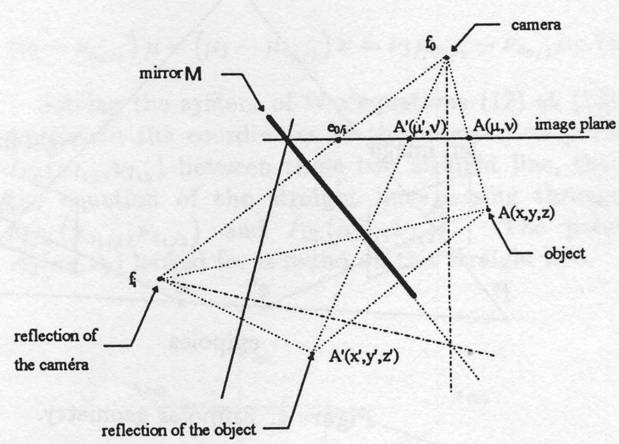

Figure 3: Example of the reflection of a point by a mirror M.

4 Epipoles and epipolar constraint

4.1 Definition of the epipoles

A point A is observed with two cameras oriented differently, of center of projection f' and f" respectively (figure 4). The point A projects on the right image plane at A' and on the left image plane at A". Knowing the point A', the corresponding point A" must be found. Actually, A' is the projection on the right image plane of the straight line passing through A and f'. A" is thus found on the projection of this straight line on the left image plane. This straight line which describes the locus of the points of the left image which can correspond to a point of the right image is called the left epipolar line. It is the intersection between the epipolar plane passing thus the points A, f' et f" and the left image plane. Identically, there exists a right epipolar line on the right image plane. All the epipolar straight lines pass through the same point, right epipole e'. This point is the image of the center of projection f" of the left camera on the right image plane. Similarly, the left epipole e" is the image of the center of projection f' of the right camera on the left image plane. The epipoles are thus the intersections between the straight line (f'f") and the image planes of the two cameras.[1][2][4][5]

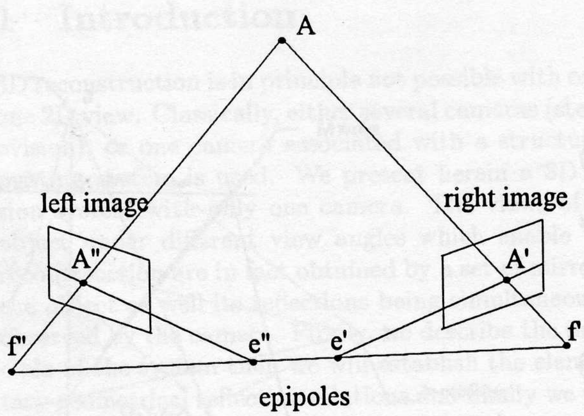

Figure 4: Epipolar geometry.

4.2 Calculating the epipoles for the system considered

Our system is composed of a camera and eight mirrors. One can consider that what one seen in a mirror M_i is equivalent to what one would see from a camera, symmetrical of the real camera by this mirror. The center of projection f_i of one of these cameras is the reflection of the center of projection f_0. The coordinates $(x_{f_i}, y_{f_i}, z_{f_i})$ can thus be determined using the relations (1). We obtain

$$\begin{aligned} x_{f_i} &= \frac{-2a_i d_i}{a_i^2 + b_i^2 + c_i^2}, \\ y_{f_i} &= \frac{-2b_i d_i}{a_i^2 + b_i^2 + c_i^2}, \\ z_{f_i} &= \frac{-2c_i d_i}{a_i^2 + b_i^2 + c_i^2}, \text{ with } i = 1, ..., 8. \end{aligned} \tag{6}$$

The epipoles $e_{0/i}$ of the central camera are the projections onto the image plane of the center of projection f_i of each virtual camera. In the image reference, the following normalized coordinates are found

$$\begin{aligned} \mu_{e_{0/i}} &= \frac{a_i}{c_i}, \\ \nu_{e_{0/i}} &= \frac{b_i}{c_i}, \text{ with } i = 1, ..., 8. \end{aligned} \tag{7}$$

Similarly, there exist epipoles $e_{j/i}$, called secondary epipoles, projections of the reflections of the centers of projection f_j by the mirror i. The coordinates of these reflections $(x_{f_{j/i}}, y_{f_{j/i}}, z_{f_{j/i}})$ in the camera's reference axes are

$$\begin{aligned} x_{f_{j/i}} &= \frac{x_{f_j}(-a_i^2+b_i^2+c_i^2) - 2a_i b_i y_{f_j} - 2a_i c_i z_{f_j} - 2a_i d_i}{a_i^2+b_i^2+c_i^2}, \\ y_{f_{j/i}} &= \frac{-2b_i a_i x_{f_j} + y_{f_j}(a_i^2-b_i^2+c_i^2) - 2b_i c_i z_{f_j} - 2b_i d_i}{a_i^2+b_i^2+c_i^2}, \\ z_{f_{j/i}} &= \frac{-2c_i a_i x_{f_j} - 2c_i b_i y_{f_j} + (a_i^2+b_i^2-c_i^2) z_{f_j} - 2c_i d_i}{a_i^2+b_i^2+c_i^2}, \\ &\text{with } i, j = 1, ..., 8. \end{aligned} \tag{8}$$

In calculating the normalized coordinates, we obtain

$$\begin{aligned} \mu_{e_{j/i}} &= \frac{x_{f_{j/i}}}{z_{f_{j/i}}}, \\ \mu_{e_{j/i}} &= \frac{x_{f_j}(-a_i^2+b_i^2+c_i^2) - 2a_i b_i y_{f_j} - 2a_i c_i z_{f_j} - 2a_i d_i}{-2a_i c_i x_{f_j} - 2b_i c_i y_{f_j} + z_{f_j}(a_i^2+b_i^2-c_i^2) - 2c_i d_i}, \\ \nu_{e_{j/i}} &= \frac{y_{f_{j/i}}}{z_{f_{j/i}}}, \\ \nu_{e_{j/i}} &= \frac{-2b_i a_i x_{f_j} + y_{f_j}(a_i^2-b_i^2+c_i^2) - 2b_i c_i z_{f_j} - 2b_i d_i}{-2a_i c_i x_{f_j} - 2b_i c_i y_{f_j} + z_{f_j}(a_i^2+b_i^2-c_i^2) - 2c_i d_i}, \\ &\text{with } i, j = 1, ..., 8. \end{aligned} \tag{9}$$

5 Stereoscopic relations

5.1 Real-reflection point mapping

An object placed at the center of the mirrors is observed with the camera. For each point $A(\mu_a, \nu_a)$ of the central image we will try to find correspondences with the images of the object on each mirror. The image point $A_i(\mu_i, \nu_i)$, reflection of the point under study by the mirror M_i must be found. To limit the search, we will use the epipolar constraint seen above, that is we are going to determine the equation of the epipolar straight line on which the reflected point is found (figure 5) [3][8][9][11].

This straight line is the image of the straight line (f_0A) by the mirror M_i. On the image, it passes through the point $e_{0/i}$ and through the point $A(\mu_a, \nu_a)$. Its equation is

$$(\nu_{e_{0/i}} - \nu_a)\,\mu - (\mu_{e_{0/i}} - \mu_a)\,\nu = \nu_{e_{0/i}}\mu_a - \nu_a\mu_{e_{0/i}}, \quad (10)$$

or, writing the coordinates of the epipole as a function of the parameters of the mirror

$$(b_i - c_i\nu_a)\,\mu - (a_i - c_i\mu_a)\,\nu = b_i\mu_a - a_i\nu_a. \quad (11)$$

The point $A_i(\mu_i, \nu_i)$ is found on this straight line.

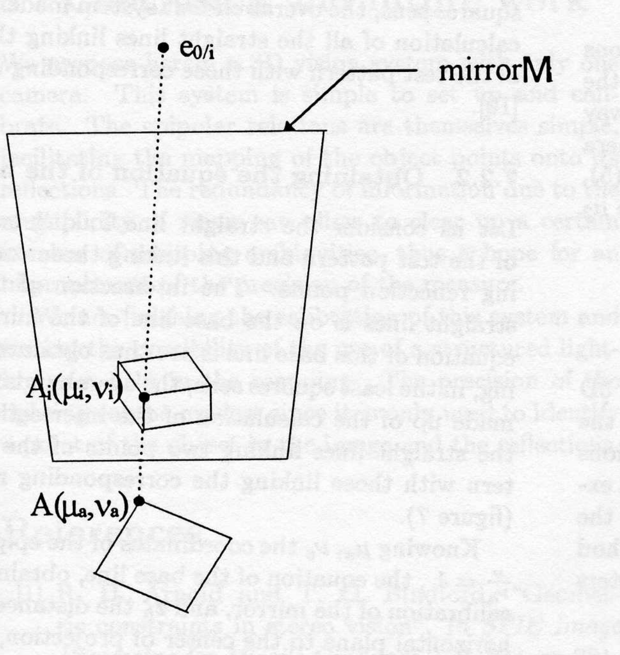

Figure 5: Object-mirror epipolar straight line.

5.2 Mapping two reflected points

We have already determined the relationship between an object point seen directly by the camera and its reflection on a mirror. But only the points found on the top side are directly visible, thus relation between two reflections on two neighboring mirrors must also be know.

If we consider two mirrors M_1 et M_2, the observed point A will have as reflection A_1 and A_2 respectively in this mirrors. Knowing the point A_1, we must find the straight line belonging to mirror M_2 in the image plane where A_2 is found.

To do so, we determine, as show in figure 6, the intersection point I_{12} between the straight line ($e_{2/1}A_1$) and the straight line defined by the intersection of the planes of the two mirrors. The point A_2 will then be located, on the image plane, on the straight line passing through the points $e_{1/2}$ and I_{12}.

Let the mirrors M_1 and M_2 of equation $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$. The equation in normalized coordinates of the image of the straight line defined by the intersection of the planes of these two mirrors is

$$(d_2a_1 - d_1a_2)\,\mu + (d_2b_1 - d_1b_2)\,\nu = (d_1c_2 - d_2c_1)\,. \quad (12)$$

The equation in normalized coordinates of the straight line passing through the point $A_1(\mu_1, \nu_1)$, supposed to be know, and the point $e_{2/1}(\mu_{e_{2/1}}, \nu_{e_{2/1}})$ is

$$(\nu_1 - \nu_{e_{2/1}})\,\mu - (\mu_1 - \mu_{e_{2/1}})\,\nu = \nu_1\mu_{e_{2/1}} - \nu_{e_{2/1}}\mu_1. \quad (13)$$

Solving the system of two equations (12) et (13), we obtain the coordinates of the intersection point $I_{12}(\mu_{I_{12}}, \nu_{I_{12}})$ between these two straight line, then the equation of the straight line passing through $e_{1/2}(\mu_{e_{1/2}}, \nu_{e_{1/2}})$ and $I_{12}(\mu_{I_{12}}, \nu_{I_{12}})$. The point $A_2(\mu_2, \nu_2)$ looked for is found on this straight line.

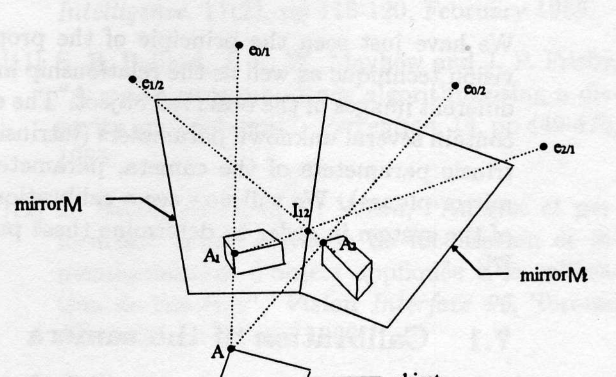

Figure 6: Mirror-mirror epipolar straight line.

6 Reconstruction

6.1 Reconstruction from the image of a point and from its reflection

Knowing the normalized coordinates of a point $A(\mu_a, \nu_a)$ and of its reflection $A_i(\mu_i, \nu_i)$ in the mirror M_i, we calculate the coordinates of the point $A(x, y, z)$ in the reference axes of the camera by solving the system of equations (2) and (5), without taking into account the variable ν_i, the latter being in fact linking to μ_i by the epipolar constraint (11). Letting a, b, c, d be the parameters of the mirror it thus become

$$\begin{aligned} Den &= \mu_a(-a^2+b^2+c^2) - \mu_i(a^2+b^2-c^2) \\ &\quad +2ac(\mu_a\mu_i - 1) - 2b(a - c\mu_i)\nu_a, \\ z &= \frac{2d(a-c\mu_i)}{Den}, \\ x &= \mu_a z, \\ y &= \nu_a z. \end{aligned} \tag{14}$$

6.2 Reconstruction from reflections on two adjacent mirrors

Knowing the normalized coordinates of the reflections $A_i(\mu_i, \nu_i)$ and $A_2(\mu_2, \nu_2)$ of the point $A(x, y, z)$ in the mirrors M_1 and M_2, we calculate in the same way the coordinates of the point $A(x, y, z)$ in the camera reference axes by solving the system of equations (5), without taking into account ν_2 since it is linking to μ_2 by the epipolar constraint.

7 Calibration

We have just seen the principle of the proposed 3D vision technique as well as the relationship among the different images of the observed object. The equations contain several unknown parameters (intrinsic and extrinsic parameters of the camera, parameters of the mirror planes). We will now see a calibration method of the system in order to determine these parameters [7].

7.1 Calibration of the camera

The camera is calibrated by the method used by C. SANTO [12][13]. This method allows to obtain the intrinsic parameters (k_u, k_v, f, u_0, v_0, distortion coefficients) and extrinsic parameters (coordinates of the center of projection and orientation of the camera, with respect to the reference plane). In the previous study, we have considered that the axis of the camera is orthogonal to the reference plane. The calibration allow to estimate the orthogonality defect which is then corrected by the method used by C. DUMONT [6]. The corrected images are thus equivalent to those obtained under the theoretical conditions described above.

7.2 Calibration of the system of mirrors

The objective is to obtain the parameters a,b,c,d of the mirrors. They are obtained thanks to the coordinates of the epipole and the equation of the base line of each mirror, the straight line defined by the intersection between the mirror and the horizontal plane.

7.2.1 Obtaining the coordinates of the epipole

Let us consider a test pattern constituted by points in a horizontal plane (figure 7). The straight line passing through the image of a point and the image of its reflection passes by the epipole. The coordinates of the epipole are thus obtained by solving, in the least squares sens, the overabundant system made up of the calculation of all the straight lines linking the points of the test pattern with those corresponding reflection [10].

7.2.2 Obtaining the equation of the base line

Let us consider the straight line linking two points of the test pattern and this linking those corresponding reflection points. The intersection of these two straight lines is on the base line of the mirror. The equation of this base line is are thus obtained by solving, in the least squares sens, the overabundant system made up of the calculation of the intersections of all the straight lines linking two points of the test pattern with those linking the corresponding reflections (figure 7).

Knowing μ_e, ν_e the coordinates of the epipole, $\frac{\mu}{\mu_h} + \frac{\nu}{\nu_h} = 1$, the equation of the base line, obtained by the calibration of the mirror, and z_h the distance from the horizontal plane to the center of projection, obtained by the calibration of the camera, the parameters of the mirror are given by

$$\begin{aligned} c &= \frac{-d}{z_h(1+\mu_h\mu_e)} = \frac{-d}{z_h(1+\nu_h\nu_e)}, \\ a &= c\mu_e, \\ b &= c\nu_e. \end{aligned} \tag{15}$$

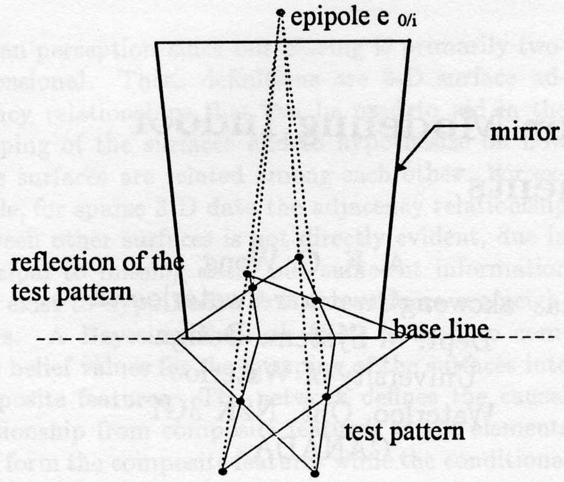

Figure 7: Calibration of a mirror.

to some arbitrary value d.

8 Conclusion and future work

We propose herein a 3D vision system with only one camera. This system is simple to set up and calibrate. The epipolar relations are themselves simple, facilitating the mapping of the object points onto its reflections. The redundancy of information due to the multiplicity of views can allow to clear up a certain number of mapping ambiguities, thus a hope for an improvement of the precision of the measure.

We are finishing the calibration of this system and testing the possibility of the use of a structured lighting as a help in the mapping. The precision of the lighting can be modest since it is only used to identify a point of the object in the image and the reflections.

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