Towards the Self-Calibration of a Multi-view Radiographic Imaging System for the 3D Reconstruction of the Human Spine and Rib Cage

F. Cheriet[1,2], J. Dansereau[1,2], Y. Petit[1,2], C. É. Aubin[1,2], H. Labelle[1], J. A. de Guise[1,3]

1. LIS3D, Research Center, Sainte-Justine Hospital, 3175 Côte Sainte-Catherine Road, Montréal, Québec, H3T 1C5, Canada.
2. Department of Mechanical Engineering, École Polytechnique, P.O. Box 6079, Station Centre-Ville, Montréal, Québec, H3C 3A7, Canada.
3. Department of Automated Production Engineering, École de Technologie Supérieure, 1100 Notre Dame Ouest Road, Montréal, Québec, H3C 1K3, Canada.

Tel: (514) 345-4931 ext 6397 Fax: (514) 345-4801
E-mail: cheriet@justine.umontreal.ca

Abstract

The main objective of this study was to develop a 3D reconstruction technique of the spine and rib cage of idiopathic scoliotic patients using the self-calibration of the imaging system. The proposed approach computes the intrinsic and extrinsic parameters of the radiographic setup with respect to the global coordinate system used at Ste-Justine Hospital. Our approach determines an optimal estimate of the geometrical parameters of the imaging system from a nonlinear minimization of the mean square distance between the observed and analytical projections of a set of matched points identified on a pair of radiographic views. The accuracy of the optimal estimate for the intrinsic parameters was significantly improved when geometric knowledge such as the known length of detectable straight bars is incorporated as a set of equality constraints in the optimization process. Furthermore, in order to retrieve the 3D structure of interest in the global coordinate system, a reference plane including the origin of the global coordinate system is specified. Computer simulations were performed to evaluate the self-calibration procedure and to determine the minimum knowledge required to obtain an accurate 3D reconstruction for clinical applications. An in vitro validation on real images of a dry cadaveric human spine showed that the method is feasible and reaches the expected accuracy.

Keywords : Self-calibration, 3D Reconstruction, Multi-view Radiographic Images, Idiopathic Scoliosis.

1. Introduction

The 3D reconstruction of the spine and rib cage from multi-view X-rays is currently done at Ste-Justine Hospital for the evaluation of the 3D deformities of idiopathic scoliotic patients. Two postero-anterior radiographs (the standard PA-0° and the twenty degrees angled down PA-20°) and one lateral radiograph are acquired for the 3D reconstruction of the spine, rib cage and pelvis. The reconstruction method requires a calibration object with known coordinates and is based on a Direct Linear Transformation technique (DLT) developed by Marzan [1] and adapted by Dansereau et al. [2]. Given the significant extrapolation error of the DLT techniques [3], the calibration object was built sufficiently large to position any anatomical structure to be reconstructed inside its limits. When the patients can be kept in the standing position during the image acquisition, the accuracy of this method is adequate for clinical 3D reconstruction of the spine and rib cage of scoliotic patients [4, 5]. However, the use of a calibration object in clinical environment presents several limitations. In fact, this object composed of two acrylic sheets located on both side of the patient is incorporated into a positioning apparatus. Therefore, the patient is constrained to be placed between these two sheets which can often create fear and inconvenience for the subject. Furthermore, the design of this object is not obvious because it must be adjustable for patients of different heights; sometimes the patient have to kneel. In spite of all these reasons, the actual design cost is not negligible and it is difficult to measure the 3D coordinates of the object with high accuracy (especially because the calibration sheets are attached separately on the positioning apparatus).

Several investigators working outside the field of medical imaging [6, 7, 8, 9, 10] have reported the basis of a technique that allows 3D object structure to be determined from biplane images obtained at arbitrary relative orientations without the use of a calibration object. Using the basic constraint of epipolar geometry applied on a minimum of eight matched points, the fundamental matrix which relates two views of the same scene is first determined. When the relative orientation of one view with respect to the other is determined, it is possible to reconstruct 3D objects only from matched points but such reconstruction is defined up to a collineation, i.e. a projective transformation. In fact, Faugeras [11] reported that computer vision may have been slightly overdoing it in trying at all costs to obtain metric information from images. In robotic applications for instance, only relative information is usually needed, so accurate metric information is not often necessary. However, in clinical practice, a reliable record of geometric changes in a global coordinate system is often required [12]. In fact, 3D reconstruction in the global coordinate system allows to derive absolute measurements for any comparative assessment of the morphology and/or the functionality of the organ of interest.

On the other hand, some authors [13, 14] have shown that the existing linear algorithms exhibit various high sensitivities to noise. It is shown that even a small pixel-level perturbation may override the information characterized by the epipolar constraint which is a key constraint used in determining 3D structure by linear algorithms.

We have described elsewhere [15, 16] a nonlinear method for the 3D reconstruction of the coronary arteries from biplane X-ray angiograms without using a calibration object. This approach made explicit use in the description of the perspective projection with the geometrical parameters of the biplane imaging system, to derive nonlinear equations that relate 3D structure with its projection in the image plane. In this application, we have taken advantage of the measurements registered from the gantry control circuit of the biplane angiographic system to choose reasonable initial guess for the geometrical parameters. Afterwards, an optimal estimate of the absolute geometry of the X-ray imaging system is determined by an iterative procedure, which minimizes the mean square distance between observed and predicted projections of a set of artery branching points identified by the clinician on a simultaneous pair of images.

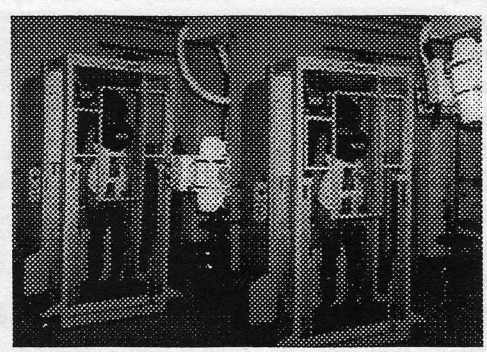
However, the imaging system used at Ste-Justine Hospital is manually positioned by a technician so it is difficult to choose reasonable initial guess for the geometrical parameters. Thus, the self-calibration of the radiographic imaging system using only the projection of matched points in a pair of images can be considered as the most general model where all parameters are free and unknown. Enciso et al [17] have shown that such problem is under-constrained.

The objective of this paper is to test the feasibility of the self-calibration of the radiographic imaging system for the 3D reconstruction of the spine and rib cage, if a prior knowledge of a minimum of information is incorporated as a set of equality constraints in the optimization process. The theoretical basis of the proposed approach is presented and the feasibility of the method is demonstrated using simulation tests and real images of a dry cadaveric human spine.

2. Description of the current radiographic imaging system

2.1 Radiographic images

The standard views used at Ste-Justine Hospital to evaluate the 3D geometry of the spine and rib cage of idiopathic scoliotic patients are the two postero-anterior radiographs obtained from the radiographic setup shown in Figure 1. The first view is a conventional PA view (PA-0°), while the second is obtained with the X-ray tube lifted up and angled down at 20° from the horizontal (PA-20°).

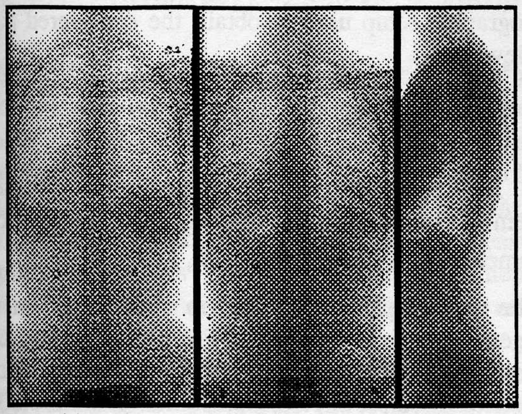


(a) (b)

Figure 1. Radiographic setup used for the postero anterior views. (a) PA-0° view; (b) PA-20° view.

A lateral radiograph can also be acquired in orde to improve the accuracy of the 3D reconstruction of th spine and pelvis. This geometrical arrangement of th

X-ray sources is a compromise to keep the standard clinical P-A view, to maximize the visibility of all radiographic features in the films (especially to see both side of the rib cage) and to keep a reasonably large stereo base for measurement accuracy [18]. Figure 2 shows the PA-0°, PA-20° and the lateral radiographs of a young adult taken with the radiographic imaging system of Ste-Justine Hospital.



(a) (b) (c)

Figure 2. Radiographic images of a young adult. (a) PA-0° radiograph, (b) PA-20° radiograph. (c) Lateral radiograph.

2.2 Calibration object

The calibration object is composed of two acrylic sheets (A and B) parallel to the X-ray film plane. The two sheets contain embedded radiopaque spherical markers (steel balls) which are easily detectable on the radiographic images (Figure 2). Figure 3 presents a scheme of the calibration object incorporated to the positioning apparatus. In practice, the patient has to stand on foot prints drawn on the adjustable floor and a lateral view is first acquired. Afterwards, the patient is rotated by 90° to bring her/him to the postero-anterior position in which the PA-0° and then the PA-20° are successively acquired.

2.3 Implicit calibration

For a pair of views (*e.g.*, PA-0° and lateral views) the perspective projection of a 3D object point onto the PA-0° and lateral image planes can be modeled as two linear transformations in homogenous coordinates with 3x4 matrices M_1 and M_2 representing respectively the calibration matrices of the PA-0° and lateral views.

$$\begin{bmatrix} w_i x_i & w_i y_i & w_i \end{bmatrix}^t = M_i \cdot \begin{bmatrix} X & Y & Z & 1 \end{bmatrix}^t_{i=1,2} \quad (1)$$

where w is a scaling factor, and (x, y) are the image coordinates of a 3D point (X, Y, Z).

In practice, the 3D coordinates of each steel ball of the calibration object were measured in the global coordinate system shown in Figure 3. The steel balls are detected and matched on the considered radiographic pair of views. The DLT technique computes the elements of the matrices M_1 and M_2 by solving the linear system obtained from the relation (1), and the measured 3D and 2D image coordinates of the steel balls.

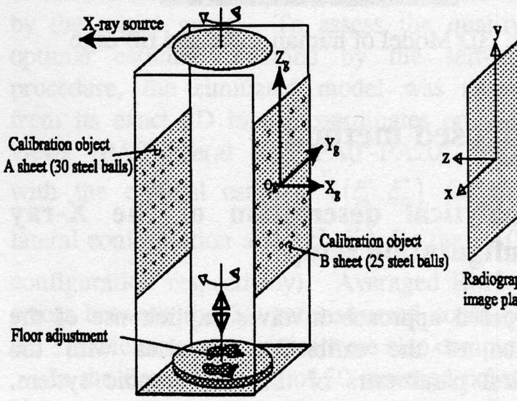


Figure 3. The calibration object incorporated to the positioning apparatus [19].

2.4 3D Reconstruction of human spine and rib cage

Once the matrices M_1 and M_2 are computed the 3D reconstruction of the bone structure from its two projections becomes the straightforward resolution of a linear set of equations. To reconstruct the spine, six anatomical landmarks per vertebra (centers of superior and inferior vertebral endplates and the tips of both pedicles) were digitized and then matched. To reconstruct the rib cage, lines were drawn on the middle of each rib image on each view. Eleven arbitrary points were marked on each of the lines on the first view then the corresponding points on the second view are deducted from the epipolar constraint. A parametric cubic spline functions were fitted to the digitized points for interpolation of intermediate points on the rib image lines. Figure 4 shows the geometric model of the spine and rib cage of a scoliotic patient resulting from the 3D reconstruction technique described by Dansereau et al. [2].

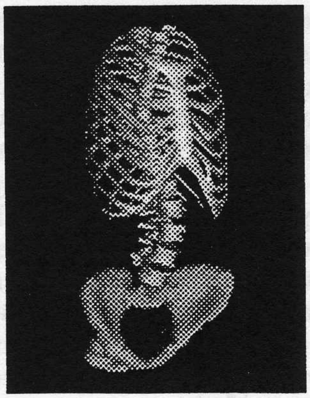


Figure 4. 3D Model of human spine and rib cage

3. Proposed method

3.1 Analytical description of the X-ray configuration setup

The proposed approach involves explicit use of the description of the calibration matrices with the geometrical parameters of the radiographic system. Thus the matrices M_i for i = 1, 3 can be decomposed as follows:

$$M_i(\xi) = P(x_{p_i}, y_{p_i}, c_{x_i}, c_{y_i}).R(\alpha_i, \beta_i, \gamma_i).T(X_{S_i}, Y_{S_i}, Z_{S_i}) \quad (2)$$

where $\xi_i = (X_{S_i}, Y_{S_i}, Z_{S_i}, \alpha_i, \beta_i, \gamma_i, x_{p_i}, y_{p_i}, c_{x_i}, c_{y_i})_{i=1,3}$ are the geometrical parameters defining the three views PA-0°, lateral and PA-20° respectively. Therefore, ten parameters are needed to describe the projection of a 3D point on one X-ray view. The matrix P represents the linear transformation associated with the perspective projection of a 3D point on the image plane. It depends on the intrinsic parameters of the system:

- x_p, y_p the coordinates of the principal point *i.e* the X-ray source projection on the image plane.
- c_x, c_y the principal distances *i.e* the magnification in the x and y directions.

The matrices R and T respectively represent the location and the orientation of the X-ray sources with respect to the global coordinate system. They depend on the extrinsic parameters of the system:

- α, β, γ the angular relationship between the image and object space coordinate systems. The matrix R is the result of sequential rotations in the specified order around the X, Y and Z axes.
- X_S, Y_S, Z_S the coordinates of the perspective center (the X-ray source) with respect to the global coordinate system.

Thus, the analytical projections of a 3D object point onto the PA-0° and lateral image planes can be modeled in homogenous coordinates as follows:

$$[w_i x_i \quad w_i y_i \quad w_i]^t = M_i(\xi_i)\cdot[X \quad Y \quad Z \quad 1]^t \quad (3)$$

where $\xi_i = (X_{S_i}, Y_{S_i}, Z_{S_i}, \alpha_i, \beta_i, \gamma_i, x_{p_i}, y_{p_i}, c_{x_i}, c_{y_i})_{i=1,2}$ represent the geometrical parameters of the radiographic setup used to obtain the considered pair of views.

3.2 Self-Calibration Algorithm

The self-calibration of the radiographic imaging system consists in computing the geometrical parameters $\xi_i = (X_{S_i}, Y_{S_i}, Z_{S_i}, \alpha_i, \beta_i, \gamma_i, x_{p_i}, y_{p_i}, c_{x_i}, c_{y_i})_{i=1,2}$ of the radiographic setup used to obtain the biplane images under investigation. For this purpose, the mean square distance between observed and analytical projections of a set of reference points of unknown 3D coordinates is minimized to provide the better estimate $\xi^* = (\xi_1^*, \xi_2^*)$. An iterative method is employed to solve this minimization problem. It requires an initial guess for each unknown parameter. Based on our experimental results, it is generally recommended to start with an initial guess near to the exact value in order to avoid ending up with a false solution. This is due to the large dimension of the parameter space and the occurrence of many local minima. For one pair of views with N reference points the parameter space is (3N+20) dimensions (20 geometrical parameters to define the pair of views and the unknown 3D coordinates of each reference point). For example, with 10 matched points the iterative algorithm has to search in a 50 dimensions space. Preliminary tests showed that a conditional estimate of parameters can be used to get a unique solution. Therefore, prior geometric constraints need to be added in order to get an optimal estimate of the system's geometrical parameters. Such constraints arise from the knowledge of the length of detectable straight bars and the specification of a reference plane including the origin of the global coordinate system. The known length of the straight bars is expressed as the relative distance measurements (d^m) between some reference pairs (p_1, p_2) in the set of matched points. The reference plane is defined with the 3D position (X^m, Y^m, Z^m) of at least 3 coplanar points. Note that one of the 3D coordinates is set to zero in order to consider a plane including the origin (0, 0, 0). This geometrical knowledge is naturally inserted in the least square approach used by the minimization process. The criterion which must be minimized to determine an optimal estimate of the geometrical parameters $\xi^* = (\xi_1^*, \xi_2^*)$ is described as follows:

$$\varepsilon(\xi) = \sum_{n=1}^{N}\sum_{i=1}^{2} d\left[\left(x_{ni}^m, y_{ni}^m\right), \left(x_{ni}(\xi_i, p_n(\xi)), y_{ni}(\xi_i, p_n(\xi))\right)\right]^2 + \sum_{k=1}^{K}\left[\left(d_k^m\right)^2 - d\left[p_1^k(\xi), p_2^k(\xi)\right]^2\right]^2 + \sum_{l=1}^{L} d\left[\left(X^m, Y^m, Z^m\right), p_l(\xi)\right]^2 \quad (4)$$

where d[*] represents Euclidean distance, (x^m, y^m) are the measured projection coordinates of the reference points on the biplane images, $(x(\xi, p), y(\xi, p))$ are the coordinates of the analytical projection of the object point p, derived from equation (3) and $p(\xi)$ is the 3D reconstruction of an object point from its measured biplane projections and the system's geometrical parameters ξ obtained at a given iteration of the minimization process.

The Levenberg-Marquardt algorithm described in [20] was chosen to compute the correction and to update the parameters at each iteration. The whole iteration process is repeated until the correction become negligible.

3.3 Simulation study

The principal goal of the self-calibration is to determine the absolute geometry of the imaging system in order to recover the 3D structure of interest from its biplane projections. To examine the inherent ability of the technique to recover 3D information, we first performed the computations under perfect conditions. To this end, a predetermined number N of object points are generated on the surface of an ellipsoid to simulate an approximate model of human trunk dimensions. The values for the system's geometrical parameters $\xi_i = \left(X_{S_i}, Y_{S_i}, Z_{S_i}, \alpha_i, \beta_i, \gamma_i, x_{p_i}, y_{p_i}, c_{x_i}, c_{y_i}\right)_{i=1,3}$ were arbitrarily selected from a set of 20 different X-ray configurations in the space of the clinical radiographic setups used at Ste-Justine Hospital. Afterwards, the mathematical projections of the simulated model are computed from the equation (3) to simulate its exact image coordinates. Thus, two sets of 20 theoretical biplane projections are generated. The PA0°-lateral pair was defined by (ξ_{1j}, ξ_{2j}) and the PA0°-PA20° was defined by (ξ_{1j}, ξ_{3j}) for j = 1,20. To choose an initial guess ξ^0 for the system's geometrical parameters, a 10% error is added to the exact value of each parameter. To assess the effect of the error introduced in the geometrical parameters on the 3D reconstruction process, the simulated model, considered as the reference, is reconstructed from its exact 2D image coordinates on the pair of views (PA0°-lateral and PA0°-PA20° respectively) with the initial guess ((ξ_1^0, ξ_2^0) for the PA0°-lateral configuration and (ξ_1^0, ξ_3^0) for the PA0°-PA20° configuration respectively). Averaged RMS distance errors between the 3D reconstructed coordinates and the 3D reference coordinates were computed. Afterwards, the self-calibration algorithm was applied to correct the erroneous geometrical parameters given by the initial guess. To assess the quality of the optimal estimate obtained by the self-calibration procedure, the simulated model was reconstructed from its exact 2D image coordinates on the pair of views (PA0°-lateral and PA0°-PA20° respectively) with the optimal estimate ((ξ_1^*, ξ_2^*) for the PA0°-lateral configuration and (ξ_1^*, ξ_3^*) for the PA0°-PA20° configuration respectively). Averaged RMS distance errors between the 3D reconstructed coordinates and the 3D reference coordinates were also computed.

In the first step, a set of 30 matched points on the biplane images and the known distance between 9 pairs of these points were used in the self-calibration procedure.

In the second step, the number of relative distance measurements and the number of coplanar points to be considered as a set of equality constraints in the optimization process have been varied to determine the minimum knowledge required to reach the best accuracy.

Finally, errors of 1 mm on the image coordinate measurements were simulated to evaluate the reliability of the technique under more realistic conditions of image quality.

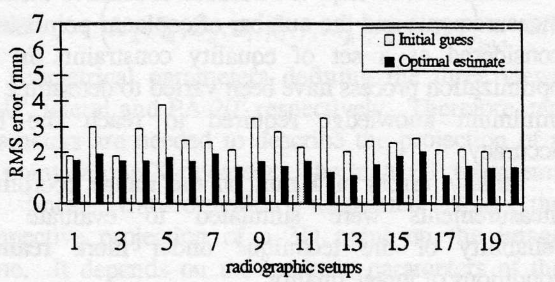
4. Results and discussion

4.1 Self-calibration under perfect conditions

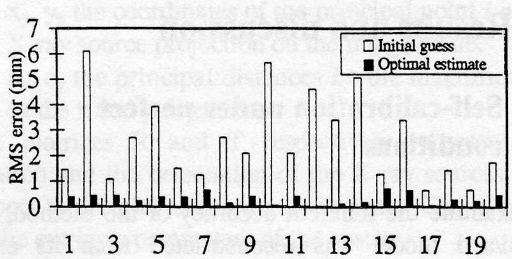
To examine the inherent accuracy of the method, the simulated model was reconstructed from its exact image coordinates on the pair of views (PA0°-lateral and PA0°-PA20° respectively) with the exact geometrical parameters ((ξ_1, ξ_2) for the PA0°-lateral configuration and (ξ_1, ξ_3) for the PA0°-PA20° configuration respectively). The RMS error was on the order of 10^{-13} mm for all the 20 radiographic setups considered. This small error is due to the floatingpoint arithmetic inaccuracy of the computer during the process of projection and back projection.

4.2 Self-calibration from 30 matched points and 9 relative distance measurements

Figure 5 shows 3D reconstruction errors of the simulated model for the 20 radiographic setups considered. Figure 5 (a) shows that the self-calibration from the PA0°-lateral pair using matched points and relative distance measurements ends up with a local minimum. The intrinsic parameters of the system were generally retrieved with a very good accuracy (up to 0.01 mm). However, the extrinsic parameters, especially the estimated position of the X-ray sources, were slightly different from the initial guesses for the 20 radiographic setups considered. Based on this experiment, it seems that information such as matched points and known distances is not sufficient to correct the errors introduced in the position of the X-ray sources with respect to the global coordinate system. The self-calibration from the PA0°-PA20° radiographic setups gave better results (Figure 5 (b), the greatest error was 0.7 mm) even if the inherent accuracy is not reached.



(a)



(b)

Figure 5. 3D reconstruction error versus radiographic setups. (a) self-calibration from PA0°-lateral pairs, (b) self-calibration from PA0°-PA20° pairs.

4.3 Minimum knowledge required for self-calibration

In this experiment the worst case of the previous section was considered (*i.e* the self-calibration from the PA0°-lateral pair). Only 18 matched points (9 pairs) were considered and the distances between these pairs were known. In addition, the 3D coordinates of control points were given to the self-calibration procedure. Figure 6 (a) shows the averaged 3D reconstruction errors, over the 20 radiographic setups considered, of the simulated model when the 3D coordinates of one, two and three points are considered. It is clear that the RMS errors, as well as their standard deviation, substantially decreased when 3D coordinates of only one control point were imposed (for instance the origin of the global coordinate system). Note that if no control point is considered the RMS errors were greater than in the previous experiment. This was expected, since only 18 matched points have been considered in this experiment (comparatively to 30 in the previous simulation). For N matched points we have 4N equations and 20 + 3N parameters (20 geometrical parameters to define a pair of views and the 3D coordinates of each matched point). By considering M known distances, the number of equations becomes 4N + M. With 9 known distances and 18 matched points, the system is over-determined but the 3D reconstruction errors as well as their standard deviations remain relatively large. In fact, the known distance measurements have improved the accuracy of the intrinsic parameters but have no effect on the determination of the 3D position of the X-ray sources with respect to the global coordinate system. However, when the 3D coordinates of three coplanar points were imposed the averaged RMS error was in the order of 10^{-13} mm which is the inherent accuracy of the method. By considering the known 3D coordinates of three control points the number of equations becomes 4x3 + 4N + M while the number of parameters is still 20 + 3N. Figure 6 (b) shows that if 3 reference points were considered, 4 relative distance measurements were sufficient to obtain the inherent accuracy of the method.

Finally, uniformly distributed errors of 1 mm (3 pixels) were introduced into the image coordinates of both views to assess the reliability of the technique under more realistic conditions. Averaged 3D reconstruction errors of the simulated model were about 2 mm which is comparable with the results obtained from the validation study of the DLT technique presented in [5]. Experiments are still underway to further evaluate the noise sensitivity of the algorithm and to determine the amount of redundancy needed in the incorporated knowledge for a robust estimation. In a long term, the robustness of the self-calibration technique can be improved by matching high-level features such as lines and edges rather than ambiguous points.

The proposed approach was also validated on real images of a dry cadaveric human spine. The inputs of the algorithm were the planar coordinates of ten balls on the B sheet (Figure 3) and ten measured distances between successive balls on both sheets A and B. The averaged 3D reconstruction error of the structure from its two projections (PA-0°, PA-20°) were 8 mm in the X direction, 1.5 mm in the Y direction and 2 mm in the Z direction. The significant error in the X direction is due to the distribution of the measured distances on only two parallel sheets A and B in the X direction.

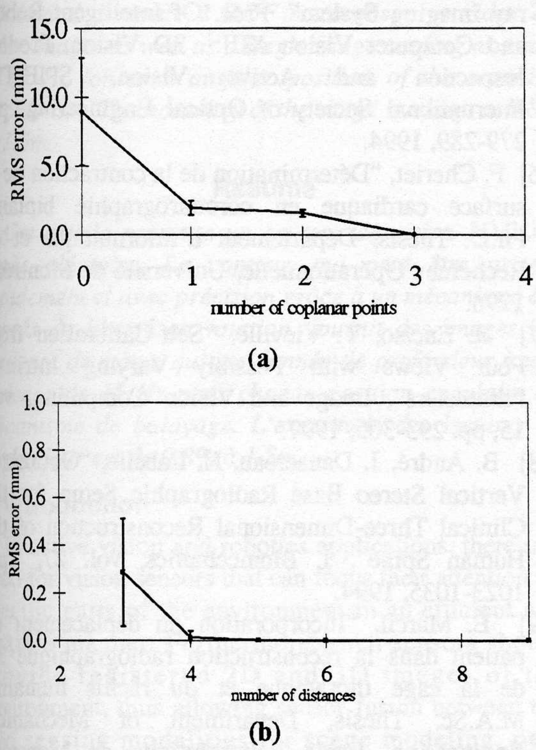


Figure 6. Determination of the minimum knowledge required for the self-calibration from the PA0°-lateral pair. 3D reconstruction errors and their standard deviations when: (a) 9 measured distances were known and coplanar points were added. (b) 3 coplanar points were imposed and measured distances were added.

In practice, our method will use a set of radiopaque straight bars of known length fixed on the patient (well distributed in the 3D space) during the radiograph acquisitions. The straight bars will be detected on the acquired multi-view radiographs and their end points will be matched on the pair of views. Then, for M bars the 2D image coordinates of 2M matched points can be measured with a reasonable accuracy. In addition, suitable control markers would be used to define a reference vertical plane of the global coordinate system. Therefore, the self-calibration approach will not require the two sheets incorporated into the positioning apparatus. Hence, the patient will be freed of the positioning device and will be allowed to adopt a normal attitude without any constraint. Also, the method will not require external 3D measurements of a calibration object. Another advantage of the proposed approach is that patient displacements occurring during the sequential acquisition of the radiographs will be compensated by equivalent orientation changes of the X-ray sources during the self-calibration process. Moreover, this technique will allow to evaluate 3D deformities of patients confined to a wheelchair and patients lying down in prone position during a surgical procedure which was not possible with previous techniques requiring a large calibration object.

5. Conclusion

This study demonstrates the feasibility of an adequate 3D reconstruction method which can be applied to reconstruct the spine and rib cage of idiopathic scoliotic patients using the self-calibration procedure of the imaging system. Prior knowledge such as at least 4 detectable straight bars and the specification of the position of at least 3 control points on a reference plane, was incorporated in order to retrieve the 3D structure of interest in the global coordinate system used at Ste-Justine Hospital. We believe that this approach will be useful and less restrictive than standard calibration techniques to evaluate the 3D deformities of idiopathic scoliotic patients.

Acknowledgements

This research was supported by the Natural Sciences & Engineering Research Council of Canada (NSERC) and Fonds pour la formation de Chercheurs et l'Aide à la Recherche du Québec (FCAR).

References

[1] G. T. Marzan, "Rational Design for Close-Range Photogrammetry", Ph.D. Thesis, Dept. of Civil Eng., Univ. Of Illinois at Urbana-Champaign, 1986.

[2] J. Dansereau and I. A. F. Stokes, "Measurements of the Three-Dimensional Shape of the Rib Cage", Journal of Biomechanics, vol. 21, pp. 893-901, 1988.

[3] G.A. Wood, R.N. Marshall, "The Accuracy of DLT Extrapolation in Three-Dimensional Film Analysis", Journal of Biomechanics, vol. 19, no. 9, pp. 781-785, 1986.

[4] J. Dansereau, A. Beauchamp, J. A. De Guise, H. Labelle, "Three-Dimensional Reconstruction of the Spine and Rib Cage from Stereoradiographic and Imaging Techniques", 16th Conference of the Canadian Society of Mechanical Engineering, Toronto, Canada, vol. 2, pp. 61-64, 1990.

[5] C. É. Aubin, J. Dansereau, F. Parent, H. Labelle, J. A. de Guise, "Morphometric Validations of Personalized 3D Reconstructions and Geometric Models of the Human Spine", Medical & Biological Engineering & Computing, vol. 38, pp. 1-8, 1997.

[6] R. M. Haralick, H. Joo, C. N. Lee, X. Zhuang, V. G. Vaidya, M. B. Kim, "Pose Estimation from Corresponding Point Data", IEEE Transactions on Systems, Man, and Cybernetics, vol. 19, no. 6, pp. 1426-1445, 1989.

[7] H. P. Trivedi, "A Semi-Analytic Method of Determining Stereo Camera Geometry from Matched Points in a Pair of Images: Coincident Meridional Planes, Exact or Noisy Data", Computer Vision, Graphics, and Image Processing, vol. 51, pp. 299-312, 1990.

[8] S. J. Maybank, O. D. Faugeras, "A Theory of Self-Calibration of a Moving Camera", International Journal of Computer Vision, vol. 8, no. 2, pp. 123-151, 1992.

[9] R. Hartley, "Self-Calibration of Stationary Cameras", International Journal of Computer Vision, vol. 22, no. 1, pp. 5-23, 1997.

[10] Q. T. Luong, O. D. Faugeras, "Self-Calibration of a Moving Camera from Point Correspondences and Fundamental Matrices", International Journal of Computer Vision, vol. 22, no. 3, pp. 261-289, 1997.

[11] O. D. Faugeras, "What Can be Seen in Three Dimensions with an Uncalibrated Stero Rig?", Proc. 2nd European Conference on Computer Vision, vol. 588 of Lecture Notes in Computer Science, pp. 563-578, Santa Marghérita Ligure, Italy, 1992.

[12] H. Labelle, J. Dansereau, C. Bellefleur, B. Poitras, "Three-Dimensional Effect of the Boston Brace on the Thoracic Spine and Rib Cage", Spine, vol.21, no. 1, pp. 59-64, 1996.

[13] C. Chatterjee, V. P. Roychowdhury, E. K. P. Chong, "A Nonlinear Gauss-Seidel Algorithm for Noncoplanar and Coplanar Camera Calibration with Convergence Analysis", Computer Vision and Image Understanding, vol. 67, no. 1, pp. 58-80, 1997.

[14] J. Weng, N. Ahuja, T.S. Huang, "Optimal Motion and Structure Estimation", IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 15, no. 9, pp. 864-884, 1993.

[15] F. Cheriet, J. Meunier, J. Lespérance, M. Bertrand, "Self-Calibration of a Stereo System Using Matched Points: Application to Biplane X-ray Imaging System", Proc. Of Intelligent Robots and Computer Vision XIII: 3D Vision, Product Inspection and Active Vision, SPIE-The International Society of Optical Engineering, pp. 279-289, 1994.

[16] F. Cheriet, "Détermination de la contraction de la surface cardiaque en coronarographie biplan", Ph.D. Thesis, Département d'Informatique et de Recherche Opérationnelle, Université de Montréal, 1996.

[17] R. Enciso, T. Vieville, "Self-Calibration from Four Views with Possibly Varying Intrinsic Parameters", Image and Vision Computing, vol. 15, pp. 293-305, 1997.

[18] B. André, J. Dansereau, H. Labelle, "Optimized Vertical Stereo Base Radiographic Setup for the Clinical Three-Dimensional Reconstruction of the Human Spine", J. Biomechanics, vol. 27, pp. 1023-1035, 1994.

[19] É. Marcil, "Incorporation du déplacement du patient dans la reconstruction radiographique 3D de la cage thoracique et du rachis humain", M.A.Sc. Thesis, Department of Mechanical Engineering, École Polytechnique de Montréal, 1996.

[20] D.W. Marquardt, "An Algorithm for Least-Squares Estimation of Nonlinear Parameters", J. Soc. Indust. Appl. Math., vol. 11, no. 2, pp. 431-441, 1963.