# Extraction of Superquadrics from Cylindrical Data Using Genetic Algorithm

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## Abstract

Recently, many models were proposed for representation 3-D model. These are applied to data compression and 3-D object recognition . Superquadrics can represent various shapes with a small number of parameters.

In this paper, we propose a method of shape description of 3-D objects from the measured range data. First we defined the error function base on the position vector and the normal vector between the measured data and superquadrics. There is a problem that the results became the local minimum, when we extract parameters of the superquadrics. In order to improve this local minimum problem, we used the Genetic Algorithm.

We apply the proposed method for assuming parameters of superquadrics from cylindrical data, and we present some experiment results of both synthetic and real range data.

## 1 Introduction

Extraction of 3-D model is very important issue of the computer vision and the robot vision research. Because of 3-D object models are necessary to recognize the shape of the objects. Now we can measure the shape of the object exactly, due to the development of the 3-D measurement technology, but usually the data measured by the 3-D measurement system are the set of the 3-D points. This data quantity is large, and it is not suitable to send and store the data. It is difficult to recognize how shape, where, how pose is object. If computers can recognize these informations of the object, it can easily understand this object independent human control. Thus, we need to represent the simple models from the measured data.

In these days, many models were proposed in order to represent 3-D model. In this paper, we used the superquadrics which can represent many shapes with a small number of parameters, but it is difficult to find the model parameters analytically for recover the model from the measured data. We defined it the optimization problem to find the parameters, and we solved this problem to use Genetic Algorithm(GA). Furthermore, we defined the energy function which uses normal vector and position vector between the measured data and the model. We propose the method of finding the parameters of the superquadrics from the cylindrical data. We show the effectiveness of our method with some experimental results.

## 2 Superquadrics

### 2.1 Concept of Superquadrics

Superquadrics(SQ) is defined a surface function which is extended on two degree surface. It can represent many shapes with a small number of parameters [1][2].

The SQ which center is in the origin of the $(x,y,z)$ space is represented with normal vector and position vector as follows

$$\vec{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a_1 \cos^{\varepsilon_1}\eta \cos^{\varepsilon_2}\omega \\ a_2 \cos^{\varepsilon_1}\eta \sin^{\varepsilon_2}\omega \\ a_3 \sin^{\varepsilon_1}\eta \end{bmatrix} \quad (1)$$

$$\vec{N} = \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = \begin{bmatrix} \frac{1}{a_1}\cos^{2-\varepsilon_1}\eta \cos^{2-\varepsilon_2}\omega \\ \frac{1}{a_2}\cos^{2-\varepsilon_1}\eta \sin^{2-\varepsilon_2}\omega \\ \frac{1}{a_3}\sin^{2-\varepsilon_1}\eta \end{bmatrix} \quad (2)$$

In the previous definition, $a_1$, $a_2$ and $a_3$ scale parameters on $x$, $y$, $z$-axises each $\varepsilon_1$ and $\varepsilon_2$ are shape parameters. We can represent many shapes easily by changing these 5 parameters. Fig.1 shows an example of changing shape parameters.

## 2.2 Transformation function of SQ

Indicated eq.(1) can represent only symmetrical shapes of $x$-$y$ plane, $y$-$z$ plane and $z$-$x$ plane. To improve the ability of the SQ representation, we use the tapering transformation. Tapering transformation is shown as follows:

$$\begin{cases} X = Ax \\ Y = By \\ Z = z \end{cases} \quad (3)$$

$$\begin{cases} A = 1 + \frac{t_1}{a_3} z \\ B = 1 + \frac{t_2}{a_3} z \end{cases} \quad (4)$$

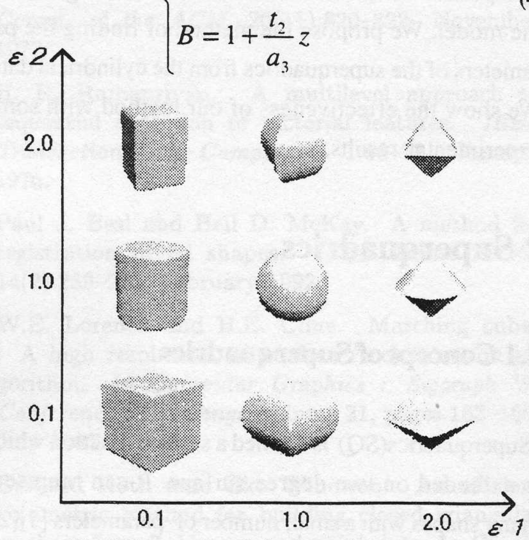
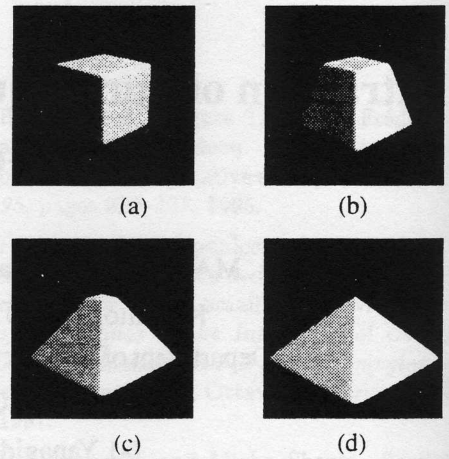



Fig.1.Example of shapes with $\varepsilon_1$ and $\varepsilon_2$




(a)$t_1=t_2=0.0$,(b)$t_1=t_2=0.3$,(c)$t_1=t_2=0.7$,(d) $t_1=t_2=1.0$

Fig.2. Example of shapes with $t_1$ and $t_2$($\varepsilon_1=0.1$,$\varepsilon_2=1.0$)

Where $(x,y,z)$ are before tapering transformation ,$(X,Y,Z)$ are after tapering transformation. $(t_1,t_2)$ are tapering parameters. These parameters have the value between -1 and 1. Fig.2 shows an example of changing tapering parameters.

To extend the original SQ equation apply to previous transformation. We can represent non-symmetric shapes using this transformation. In addition to these 7 parameters, we used rotate parameters which represent rotation on $x$,$y$,$z$-axises and position parameters which are place position on the $x$,$y$,$z$-axises. Therefore, we need to set 13 parameters to represent one SQ shape.

# 3 Extraction of SQ

We describe about the SQ extraction method from the measured data. It is often difficult to extract SQ from the measured data observed from one direction. For example, we can not understand cylinder or cone shape measured from specific side in shown Fig.3. So we used cylindrical data in order to extract models correctly. We used cylindrical coordinate as cylindrical data. The decision of the fittest parameters using the range data is the optimization problem to find from the parameter set. In order to solve this problem, we use GA with the energy function between the SQ model and the measured data.

## 3.1 Cylindrical coordinate

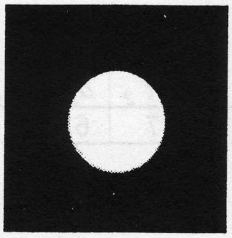
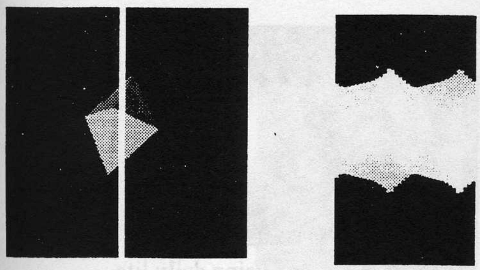


Fig.3.Difficult to recognize object



(a) (b)

(a)Diamond shape object,(b)Cylindrical coordnate

Fig.4. Translation to cylindrical coordnate

In this paper, we used cylindrical coordinate. We set rotation axis passing object. The horizontal axis is defined rotation value between 0 and $2\pi$. The data is measured by rotating the rotation axis. The vertical is axis's coordinate value. The coordinate's gray scale value means Euclidean distance from the rotation axis. As an example, we present diamond shape translated to cylindrical data. Fig.4(a) shows object of diamond shape. The white line in Fig.4(a) is rotation axis. Fig.4(b) shows Fig.4(a) translating to cylindrical coordinate.

## 3.2 Definition of the energy function

We define the energy function which uses normal vector and position vector between the measured data and the model as follows[3]:

$$E = E_1 + \alpha E_2 \tag{5}$$

$$E_1 = \frac{1}{p}\sum_{i=1}^{p}\left|\vec{X}_i - \vec{x}_i\right| \tag{6}$$

$$E_2 = \frac{1}{p}\sum_{i=1}^{p}\left|\vec{N}_i - \vec{n}_i\right| \tag{7}$$

Where $\alpha$ represents weight. $E_1$ and $E_2$ represent the error of the position vector and the normal vector. Furthermore, $\vec{x}, \vec{n}$ are the position vector and the normal vector of the measured data. $\vec{X}, \vec{N}$ are the position vector and the normal vector of SQ on the line which is

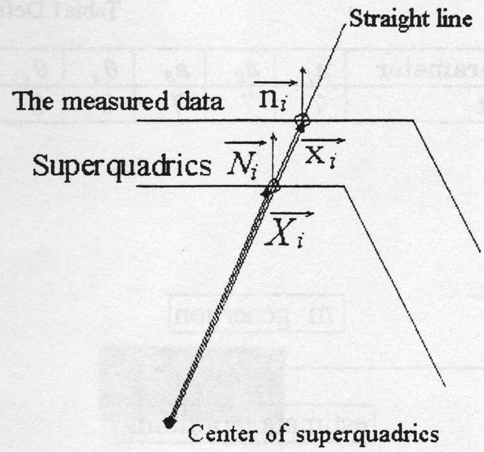



Fig.5.The pair of reference points

drawn from the measured data to center of SQ. Fig.5 shows these points relation. The position vector and the normal vector are done inverse translation. Eq.(8) and eq.(9) show this inverse translation.

$$\begin{cases} x = \frac{X}{A} \\ y = \frac{Y}{B} \\ z = Z \end{cases} \tag{8}$$

$$\begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = \begin{bmatrix} \frac{1}{B} & 0 & 0 \\ 0 & \frac{1}{A} & 0 \\ \frac{\left(\frac{r_1}{r_3}\right)X}{A^2B} & \frac{\left(\frac{r_2}{r_3}\right)Y}{AB^2} & \frac{1}{AB} \end{bmatrix} \begin{bmatrix} N_x \\ N_y \\ N_z \end{bmatrix} \tag{9}$$

## 3.3 Extraction of SQ using GA

In order to extract the SQ from the measured data, we consider that it is the optimization problem to find the parameters. We solved this problem using GA[4]. We made the string of SQ parameters by the binary and coupled them. It is effectiveness of GA operation to connect each relation parameters. Concretely, they connect the shape parameters and the rotation parameters, because their parameters contain the information to concern surface. Scale parameters connect to first line, and

Table1.Definition of strings

| parameter | $a_1$ | $a_2$ | $a_3$ | $\theta_x$ | $\theta_y$ | $\theta_z$ | $\varepsilon_1$ | $\varepsilon_2$ | $t_1$ | $t_2$ | $d_x$ | $d_y$ | $d_z$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| bit | 7 | 7 | 7 | 9 | 9 | 9 | 8 | 8 | 7 | 7 | 6 | 6 | 6 |

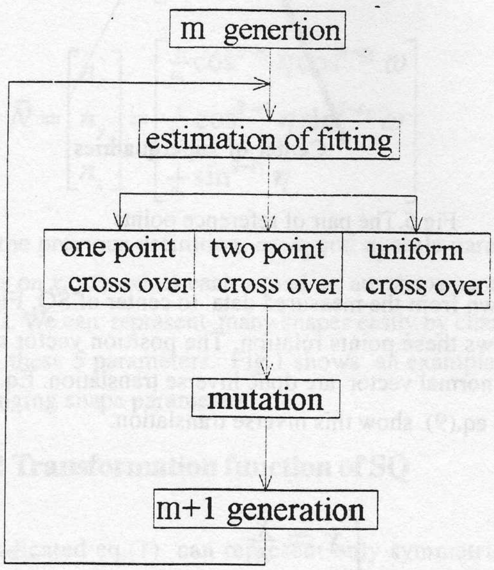



Fig.6.The flow of the evoluationary process

position parameters connect to last line. Their strings connections are shown in Table1.

First, some individuals for the first generation are generated at random. And two individuals are selected at random from the parent group. These individuals are applied one method among one point crossover, two point crossover and uniform crossover at random. They are selected at random with the same probability. Then, these individuals are applied to mutation. The individuals are selected by roulette strategy to the next generation. By repeatedly these process, the group of individuals ties up to optimum answer. Fig .6 shows the flow of GA.

We defined the parameters range as follows

$e_1, e_2$: 0.01 - 2.56

$a_1, a_2, a_3$ : 1 - 65

$d_x, d_y, d_z$ : -32 - +32

$\theta_x, \theta_y, \theta_z$ : $0 - 2\pi$

$t_1, t_2$: 0 - 1

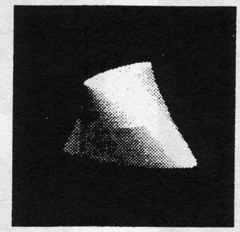


Fig.7.A rare shape using daily life

Tapering parameters have value between -1 and 1 originally, but in this paper, we are this value limited between 0 and 1. If each tapering parameters value is difference sign, SQ becomes a rare shape for using industrial parts and daily life like Fig.7. In the case of each these parameters are positive or negative numbers, they are equal to represent each abilities. So we use tapering parameters in only positive numbers.

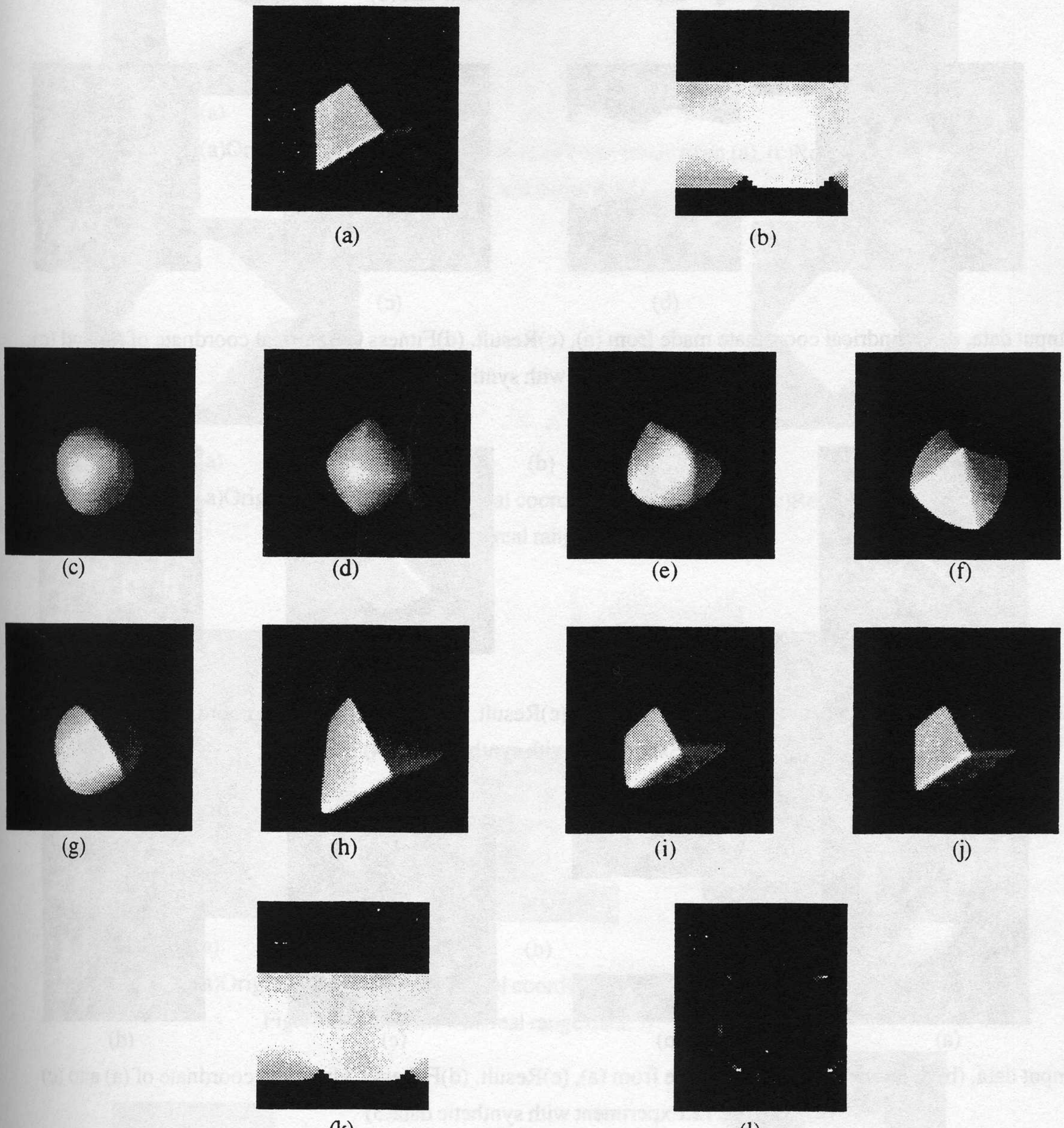
# 4 Experimental results

We present some experiments to extract SQ from the both synthetic and real range data.

## 4.1 Experimental results using synthetic data

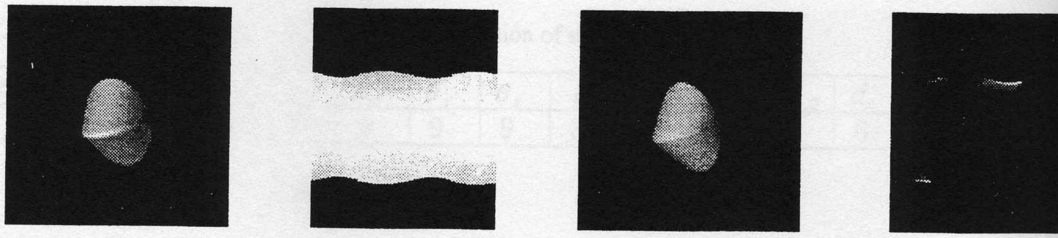
We made SQ using eq.(1). We made cylindric coordinate data. Fig.8(a) shows making synthetic data ($a_1=a_2=50, a_3=70, e_1=0.1, e_2=0.1, t_1=0.3, t_2=0.3$). Fig.8(b) shows translation Fig.8(a) to the cylindrical coordinate data.

Fig.8(c)-(i) show shapes of SQ by changing the generation. Fig.8(j) shows result of recovering SQ. Fig.8(k) shows translation Fig8.(j) to cylindrical coordinate data. Fig8.(l) shows fitness between Fig8.(b) with Fig8.(k). We could get nearly optimum answer about position, shape and pose. We experimented some shapes(Fig.9-Fig.12). We could get good results in experiments for synthetic data.




(a)Input data, (b)Cylindrical coordinate made from (a), (c)First generation, (d)20 generation, (e)50 generation, (f)100 generation, (g)200 generation, (h)500 generation, (i)800generation, (j)Result, (k)Cylindrical coordinate made from (j), (l)Fitness between (b) and (k)

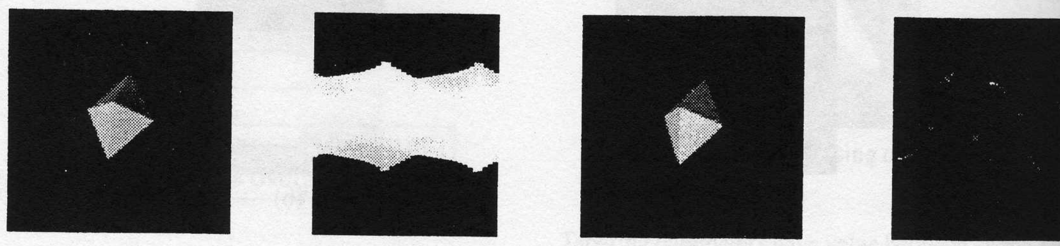
Fig.8.Experiment with synthetic data(1)



(a) (b) (c) (d)

(a)Input data, (b)Cylindrical coordinate made from (a), (c)Result, (d)Fitness Cylindrical coordnate of (a) and (c)

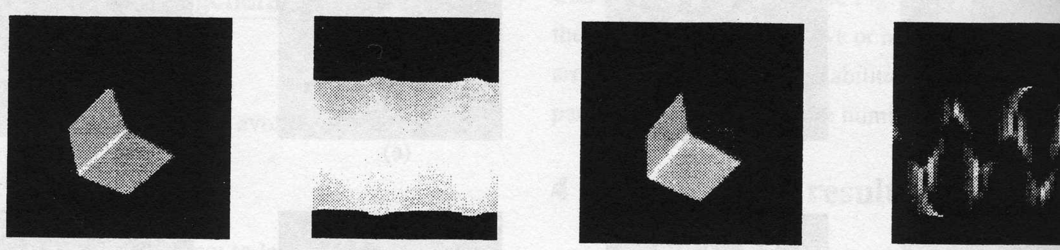
Fig.9.Experiment with synthetic data(2)



(a) (b) (c) (d)

(a)Input data, (b)Cylindrical coordinate made from (a), (c)Result, (d)Fitness Cylindrical coordnate of (a) and (c)

Fig.10.Experiment with synthetic data(3)



(a) (b) (c) (d)

(a)Input data, (b)Cylindrical coordinate made from (a), (c)Result, (d)Fitness Cylindrical coordnate of (a) and (c)

Fig.11.Experiment with synthetic data(4)

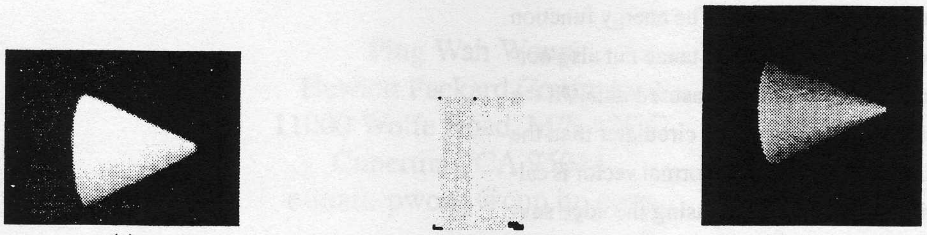
(a) (b) (c) (d)

(a)Input data, (b)Cylindrical coordinate made from (a), (c)Result, (d)Fitness Cylindrical coordnate of (a) and (c)

Fig.12.Experiment with synthetic data(5)

## 4.2 Experimental results using real range data

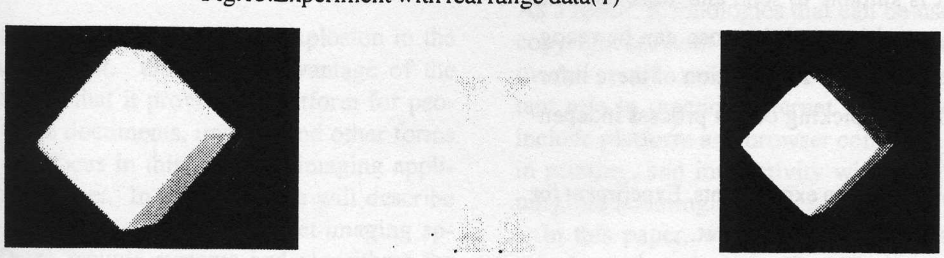
We measured 3-D data by a synchronized laser scanner (HYSCAN45C) patented NRC in Canada. The scanner can measure 3-D data error within 25 $\mu m$. We measured the cylindrical data of the object. We translated cylindrical data to cylindrical coordinate. We experimented to extract SQ from it. We experimented some shapes(Fig.13-Fig.16). We could get good results in this experiments, too.



(a) (b) (c)

(a)Original object, (b)Cylindrical coordinate made from (a), (c)Result

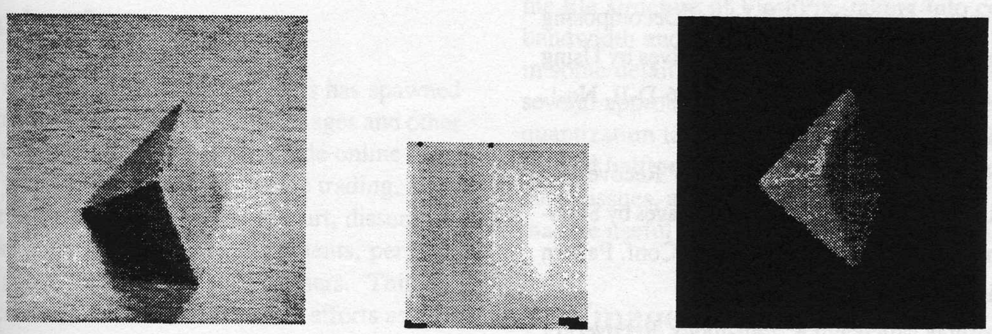
Fig.13.Experiment with real range data(1)



(a) (b) (c)

(a)Original object, (b)Cylindrical coordinate made from (a), (c)Result

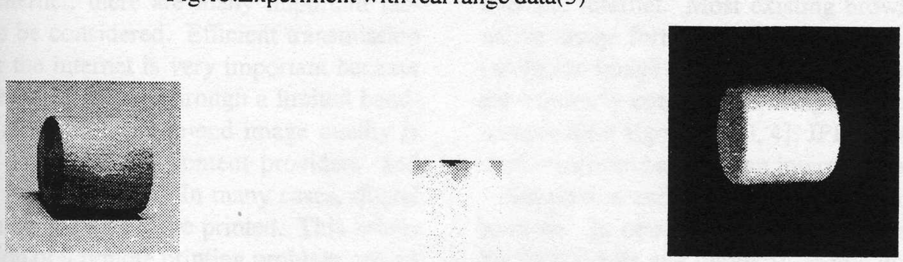
Fig.14.Experiment with real range data(2)



(a) (b) (c)

(a)Original object, (b)Cylindrical coordinate made from (a). (c)Result

Fig.15.Experiment with real range data(3)



(a) (b) (c)

(a)Original object, (b)Cylindrical coordinate made from (a), (c)Result

Fig.16.Experiment with real range data(4)

## 5 Conclusion

We propose the method for extracting SQ from cylindrical data, and experiment both synthetic and real range data. Using GA can find a global minimum avoiding local minimum of fitness function. The energy function does not include only Euclidean distance but also normal vector between SQ and the measured data. At the edges of the object, it became more circulater than the measured data. However, when the normal vector is calculated, we think result is fitter by using the edge saving method.

By using our method, we can compress the 3-D data to 13 numbers. It is suitable to send and store the data. The shape, the position and the pose can be recognized from the range data. Recognition of these informations is applied to picking object process independent human control.

We used simple objects in experiments. Experiment for complex objects is our coming subject.

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