Extraction of Handwritten Data from Noisy Gray-Level Images Using A Multi-Scale Approach

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Abstract

This paper presents an effective approach to extract hand-printed zip codes from noisy gray-level images. Since in these and other alike images the intensity changes may occur over a large range of spatial scales, we adopt the multi-scale approach to extract good data quality that might be used in further processing and recognition processes. We have already shown in a previous paper its effective use in full data extraction. In this paper, we will give an advanced formalism of this result, referred to as a top-down approach. Then, we will present a new and opposite approach, referred to as bottom-up approach. The differences and characteristics of both approaches are highlighted. To asses this study, experimental results have been obtained on real life data from the data base provided by CEDAR at SUNY Buffalo.

Key Words: image segmentation, multi-scale concept and methodology, handwritten data segmentation and recognition, document processing and recognition.

1. Introduction

Currently there is substantial and growing interest in the field of document image processing and understanding where several research groups are developing and designing systems to automatically process relevant information from documents such as engineering drawings, maps, magazines, or mail envelopes [1,2]. More generally, the problem of image segmentation has received considerable attention in the literature [3-7]. Many methodologies have been proposed to tackle this problem. However two approaches are widely used in this context : the *edge-based approach* [8], where local discontinuities are detected first and then connected to form longer, hopefully complete boundaries; and the *region-based approach* [9], where areas with homogeneous properties are found, which in turn give the boundaries.

Our interest in this study is to present a new methodology to segment visual data contained in noisy gray-level images such as mail envelopes. In fact, in automatic processing of handwritten data (fig.1), preserving the topological properties of the extracted data is important in improving a system's performance. Natural images of unconstrained handwritten data contain characters of different sizes, with variable inter-object distances; in addition, the intensity changes in these images may occur over a wide range of spatial scales. When such an image is binarized, there can be a significant information loss, resulting in broken or touching characters, and the presence of noise or unnecessary information. The intention behind the approaches presented here is to preprocess the data and to derive useful information (while reducing noise) that might be of interest for later stages of information processing. The paper deals with the fundamental problems that are associated with the use of scale-space analysis in early processing of visual information, see Figure 1.

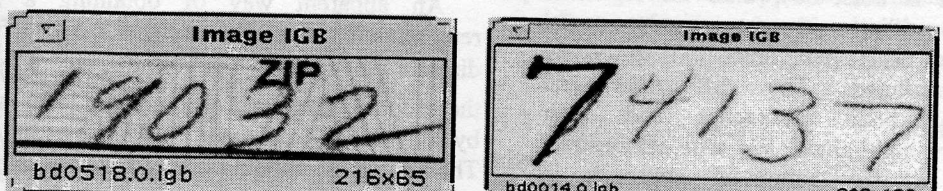


Figure 1: Example of images to be processed

2. Related work and our contribution

An intrinsic property of objects in the world and details in images is that they only exist as meaningful entities over certain ranges of scale. A multi-scale representation of data is crucial for describing the structure of unknown real-world signals.

In his book, Lindeberg [12] presented basic scale-space theory as well a thorough review of multi-scale analysis. A summary is given of basic properties of scale-space and related multi-scale representations, notably, pyramids, wavelets, and regularization. A number of special properties of the scale-space representation are listed, and the different multi-scale approaches compared. Although, the basic intention behind this framework is to handle structures at different scales (features and relation between features at different levels of scale), our motivation is to use this well-founded framework for improving visual data segmentation. In other words, the structures and the features we are interested in at different levels of scale are pixels themselves and their groupings as perceived in the scene.

Since in our application and other alike images the intensity changes may occur over a large range of spatial scales (Figure 1), we believe that the use of multi-scale approach helps in extracting good data quality that might be used in further processing and recognition processes. While conventional methods focus on the successful use of such a paradigm in edge detection, we have already shown in [6] its use for full shape data segmentation. We will refer to this method, noise-driven removing, as *a top-down* approach. Starting with the coarsest approximation at the lowest level, the signal is reassembled by adding on successively finer details at each scale-space level until the original image is reconstituted. In this paper, we will present an advanced formalism of this method; in addition, we will propose an opposite scheme that should be able to increase accuracy in visual data segmentation. We refer to the new approach, data-driven grouping, as a *bottom-up* approach. Instead of the coarse image, we start with a lower resolution (sparse data) that is noise-free, then the signal is reassembled by adding on successively useful information at each scale-space level until the visual image is reconstituted. The differences and characteristics of both approaches and their implementation are highlighted. We will demonstrate the contribution of the bottom-up approach in improving the visual shape quality of the extracted data.

Although they are opposite, both top-down and bottom-up approaches rely on one family of smoothed images, using the Gaussian kernel for generating scale space. As we will show, the iteration parameter in the former is the standard deviation, while in the latter it is the mask size. By doing this, we minimize the loss of information, and fine image quality can be drastically improved and become more accurate. More details about theoretical and qualitative aspects will be investigated in subsequent sections.

3. The Multi-scale concept overview

The basic idea is to embed the original signal into a one-parameter family of gradually smoothed signals, in which the fine-scale details are successively suppressed. It can be shown also that the Gaussian Kernel and its derivatives are singled out as the only possible smoothing kernels. Originating from the computational vision model in [13], in which zero-crossings of the Laplacian play a dominant role, substantial efforts have been spent on analyzing the information content of those features in scale-space [14-16]. There have been thorough investigations into theoretical properties of this representation. The behavior of structures under Gaussian smoothing is analyzed in [17]. Other studies concerning edges include [18-20]. A methodology proposed in [17,21] extends the scale-space concept to two-dimensional signals. This formulation can be extended to arbitrary dimension. Some related formulations have been expressed by [22,23], and [24,25] propose more extensions.

3.1 Construction of discrete scale-space

The hierarchical approach consists in starting with the coarse level of scale-space (obtained by strong blurring by the Gaussian average operator) and gradually focusing on the object of interest by varying the scale space continuously to a fine scale space. The coarse-to-fine tracking in a continuous manner combines high

$$I_i = I_{i-1} * K_i$$

accuracy with good noise reduction. The mathematical background and the general algorithm of this approach can be found in [18,25].

An apparent way of obtaining a multi-scale representation of a discrete signal I is to define a set of discrete functions I_i ($i \in [0 .. n]$), where I_0 represents the original image and each coarser level is calculated by convolution from the previous one as follows :

The kernels K_i should be appropriately selected scale-space kernels corresponding to suitable amounts of blurring. The scale-space condition for each kernel guarantees that signals at coarser levels of scale do not contain more structure (information) than signals at finer levels of scale. This indicates that the causality criterion is respected. We would like to note that homogeneity and isotropy criteria are also respected with this approach, as stated by [12].

3.2 Gaussian kernel and its Laplacian as decision criterion

As mentioned in the introduction, the Gaussian kernel is the unique kernel for generating scale-space. Since we are interested in obtaining visual frontiers of data, we consider the second derivative of such a kernel. So, we start describing the method by recalling the definition of the Laplacian of Gaussian operator (LOG), [6]. Let g(x,y) be a 2D symmetrical Gaussian distribution

$$g(x,y) = e^{-[\frac{x^2+y^2}{2\sigma^2}]}$$

Theoretically, the surface subtended by the curve within the range [-3σ, +3σ] equals 99.74%, while it the range [-4σ, +4σ] covers 99.99%. Thus, the mask size of the smoothing filter must be within the range [6σ, 8σ] in order to have a good response. Now, let ∇^2 be a 2D Laplacian operator, then, the filter is defined as convolving the image with a Laplacian of Gaussian (LOG):

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \quad \text{then}$$

$$\nabla^2 g(x,y) = \left(\frac{x^2+y^2}{\sigma^4} - \frac{1}{\sigma^2} \right) e^{-[\frac{x^2+y^2}{2\sigma^2}]}$$

Unlike the other differential operators, the LOG operator is simple, general, and is applied on the image in one step. The cross section of the LOG function is shown below.

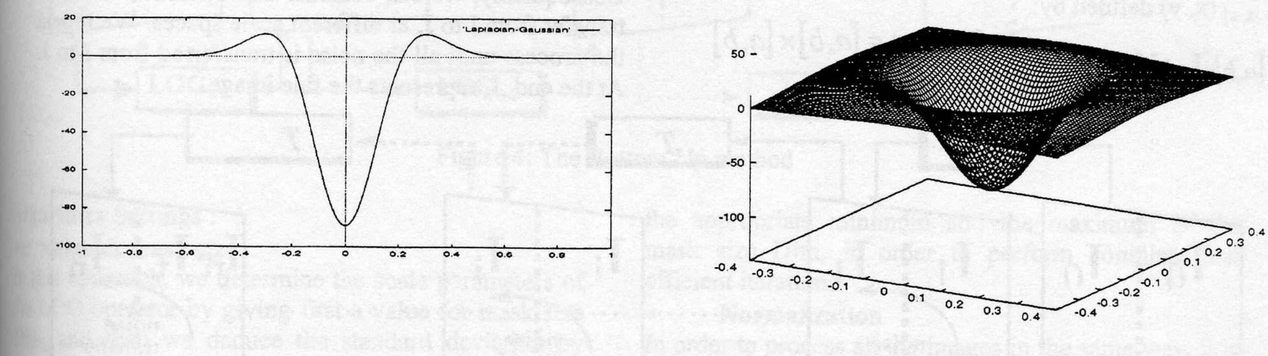


Figure 2 : one- and two-dimensional profiles of Laplacian-Gaussian (LOG). The positive bumps occur at about ± 2*σ

The main use of LOG kernel at scale space is the edge detection in natural images. The problem is that natural images are noisy and vary in contrast, leading to discontinuities in visual boundaries of objects. This happens especially when σ is large. The solution of this problem consists in thresholding the image in the laplacian scale space. However, a question remains to be answered: which σ value should be used? The σ value that seems to answer this question is approximately equal to 2. The larger the σ value is, the more smoothed the image will be. Furthermore, one of the effects of such a process is the displacement of edges to outside of the object, thereby increasing its volume. Conversely, if the standard deviation is lower than 2, the envelope of the objects shrinks and may be located inside the visual frontiers of the object. In addition, the edges will be more noisy. Thus, it should be obvious that the value of the standard deviation depends completely on the desired application.

By thresholding in Laplacian space, using the zero crossings as thresholds, we would be able to obtain accurate segmentation that fits with visual decisions. The weak regions, which are more problematic, are usually recovered in different scale spaces. The following section presents the multi-scale space based methods that we developed to segment handwritten data from noisy background. For discrete implementation of this operator, the mask size depends naturally on the value of σ. Clearly, the performance of the operator also depends on the value of σ.

3.3 Algorithm Settings : σ value & mask size

It is possible to determine the mask size that contains the LOG kernel, since more than 99% of the area delimited by the canonical normal form is within -3σ and +3σ from the origin. We will show that in the top-down approach, the parameters of LOG operator are determined by giving a value for σ and then the size mask is deduced by means of the formula: Dim = 6*σ. In the new approach we do the contrary.

4 Methods

4.1 The Top-Down Approach : Advanced Formalism

As we mentioned above, the top-down approach consists in applying LOG operator to the multi-scale image in order to obtain a coarse image of the handwritten image and repeat this process iteratively until getting the fine image. Figure 3 describes the different steps of this process. In the following, we present an advanced formalism of the approach.
The mask and the images are bounded sets of IR^2. However, the convolution is defined only on all the space (IR^2). We introduce the truncated function $1_{[a,b]\times[a,b]}(x, y)$ defined by:

$$1_{[a,b]\times[a,b]}(x,y) = \begin{cases} 1 \text{ if } (x,y) \in [a,b]\times[a,b] \\ 0 \text{ elsewhere} \end{cases}$$

by using the same notation as in section 3.1 we can write

$$I_i = \left(LOG_i \times 1_{[-dim_i, dim_i]\times[-dim_i, dim_i]}\right) \times \left(I_{i-1} \times 1_{[a,b]\times[a,b]}\right)$$

where [a, b][a, b] is the dimension of the original image. We keep the LOG notation for the truncated LOG ($LOG \times 1_{[-dim, dim]\times[-dim, dim]}$), and I for $I_{i-1} \times 1_{[a, b]\times[a, b]}$. We refer to the image and the function defining the image by the same symbol.
In Figure 3, I_0 represents the original image and J_0 an empty image (all the pixels are turned off) which will contain the extracted noise from I_0. The LOG_1 (with $\sigma = \sigma_{max}$) is applied to I_0; all the pixels with non-positive values are turned off in I_0 which give I_1, and turned on in J_0 which give J_1. This operation is performed by the operation *T*, by using the LOG operator. If we decompose $I_0 * LOG_1$ as : $I_0 * LOG_1 = I_0^+ + I_0^-$, where $I_0^+ = Sup(0, I_0 * LOG_1)$ and $I_0^- = Inf(0, I_0 * LOG_1)$; the I_0^- function of negative values gives J_1, and I_0^+ function of positive values gives I_1.
Consequently, we can consider this operation as noise transfer from I to J, at different scale spaces. We repeat the process until all the noise is transferred from I to J. At the end, I_n represents the fine image.

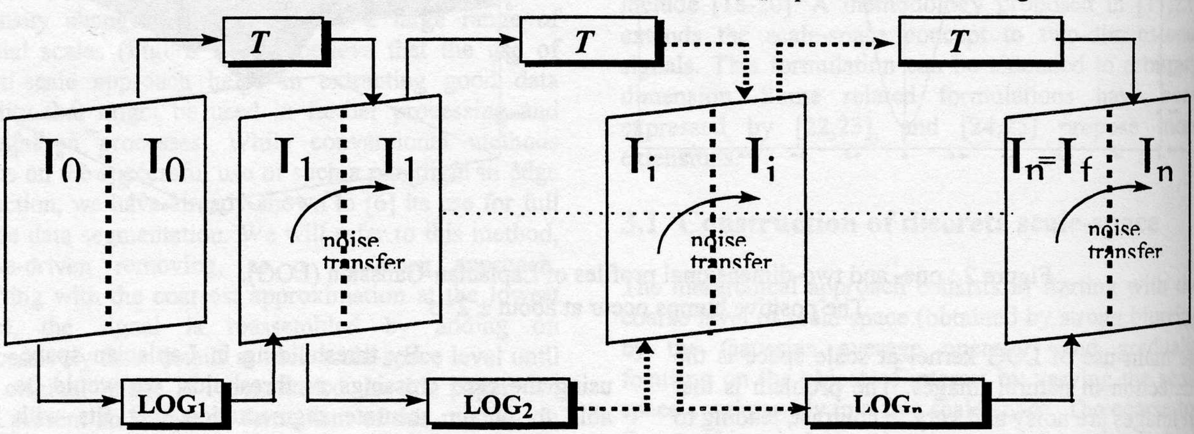


Figure 3: The Top-Down approach

The scale space parameter is the standard deviation; the mask size is therefore deduced at each step, as defined in section 3.3. Finally we should recall that this construction has been built in order to segment full shape data, and not to detect only data edges. To do so, the convex parts of the kernel operator have been taken as the decision criterion. The detailed algorithm, experimental results and discussions of this approach are detailed in [6].

4.2 The Bottom-Up Approach

The Bottom-Up method consists in applying the LOG operator to the multi-scale image in order to extract minimum information from the original image which is noise free (sparse data) and repeating this process iteratively in order to recover accurate information, at each step, until arriving at the fine image. Figure 4 depicts the different steps of this process. I_0 represents the original image and J_0 an empty image (all the pixels are turned off) which will contain the extracted information from I_0. The LOG_{n-1} (with $\sigma = \sigma_{min}$) is applied to I_0, all the pixels with positive values are turned off in I_0 which gives I_1 and turned on in J_0 which gives J_1; this operation is performed by *T*. Analogously we can formally write : $I_0 * LOG_{n-1} = I_0^+ + I_0$ where I_0^+

$= Sup(0, I_0 * LOG_{n-1})$ et $I_0^- = Inf(0, I_0 * LOG_{n-1})$; the I_0^- function of negative values gives I_1, and I_0^+ function of positive ones gives J_1.

We can see this operation as an information transfer from I to J. Thus, we repeat the process until all the information is transferred from I to J. In this method, two supplement steps are applied. First, all the shapes within the image J_N are identified as connected components (Shape Identification Step), and then are matched with those identified in J_1 image (noise free and called a uni-resolution image). Only if the matching is empty for a given shape should it be considered as a noise and removed (Noise Removal Step). The fine image is represented by J_f. Because of interpreting the process as a transfer of information from I to J, there is no new information that is created at any scale space, which means that the causality principle is respected. Furthermore, homogeneity and isotropy are also respected (we use the same process as in the top-down approach).

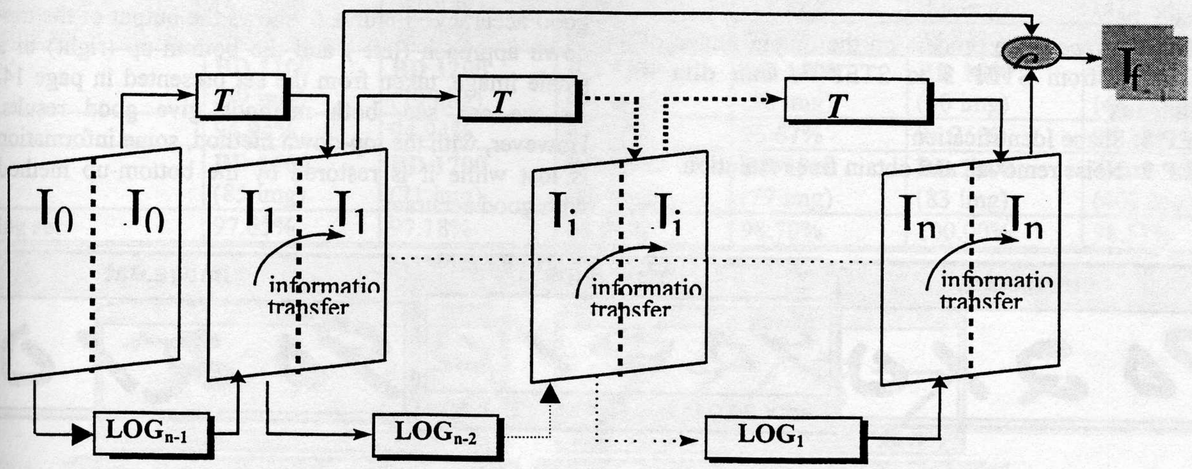


Figure 4: The Bottom-Up method

Algorithm Settings :

•σ value & mask size

In this approach, we determine the scale parameters of the LOG operator by giving first a value for mask size Dim, and then we deduce the standard deviation by means of the formula: Dim = 6*σ. This way enables us to have accurate shapes when convolving the image with the kernel.

- **Variation coefficient parameter (Relation between object size and selected scale)**

All the images do not necessarily require the same level of resolution. Furthermore, the scale value does not necessarily reflect the size of the blob region in the image. Although large values of the scale parameter in general will lead to images with large features, there is no direct relationship between the size of an object and its selected scale. Hence, we use the variation coefficient (VC) as an image characteristic. Since the VC is a ratio of the mean over the standard deviation, it allows us to obtain information about image homogeneity. If the VC is very low, (*e.g.* < 0.195 in our application), all the pixels have sensibly the same intensity, thus we have to apply a transfer function, *e.g.* the squared transfer function. Conversely, if VC is high (> .49), the handwriting is thick and dark; we should then begin with an adequate level of resolution. Finally, the VC value for a given image allows us to compute the appropriate minimum and the maximum of the mask size Dim, in order to perform complete and efficient iterations.

- **Normalization**

In order to process all the images in the same way, it is necessary to normalize them by an homothetic transformation. Thus all the pixels of an image are extrapolated between 0 and 255.

$$I(x,y) = (I(x,y) - min)\left(\frac{255}{max - min}\right)$$

Where min and max are respectively the minimal and maximal luminance values of the image.

We present now the general algorithm for the bottom-up approach:

Procedure Multi_Resolution_BU( )
Begin

STEP 0 : Normalize the image. Compute the VC, then apply a transfer function if necessary.

STEP 1 : reach pixel of the grey level image, apply the LOG kernel with a step scale, **dim = DimMin**, sufficiently low to extract a minimum amount of information (sparse data) free of noise; then, compute the corresponding σ value.

STEP 2 : Threshold the output image by turning on pixels with negative values, and turning off those with positive or null values.

STEP 3 : Turn on the neighbors of pixels that are the output of step.

STEP 4 : Increase the mask dimension.

STEP 5 : Apply the LOG kernel to the original image with the step scale, only on pixels being turned on at the previous steps.

STEP 6 : Threshold the output image as in step 2. The output image constitutes the desired result at the scale space .

STEP 7 : *Repeat* the process on the output image of STEP 6, from STEP 3 to STEP 7, *until* **dim** = **DimMax**

STEP 8: Shape Identification.

STEP 9: Noise removal, and obtain fine resolution.

End

5. Comparison of the methods

Considering the top-down approach, we may face the following problem: some information may be transferred with noise. So, there is no way to retrieve the lost information. If we consider the bottom-up approach, we may be confronted with an analogue problem, namely, some noise may be transferred along with the information. However in the latter, the shape identification step conjugated with the noise removal process, assure us an efficient segmentation with a good accuracy. Figures 5 shows the output of the top-down approach (left) and the bottom-up (right) of a smale image, taken from the set presented in page 14. As we can see, both methods give good results. However, with the top-down method, some information is lost while it is restored by the bottom-up method with good accuracy.

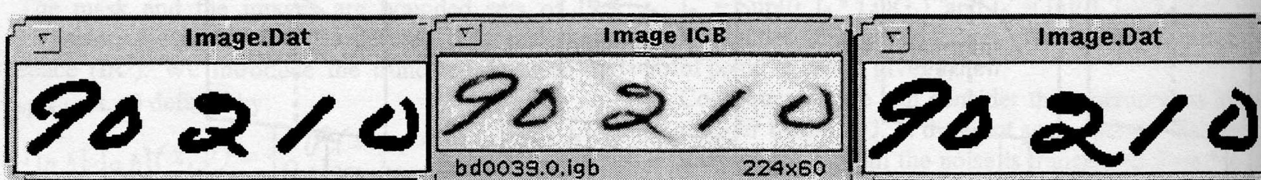


Figure 5 : Results of both top-down (left) and bottom-up (right) approaches on a same sample image

6. Results of the Bottom-up method

As pointed out in section 2, the purpose of the new proposed thresholding method is to develop a general and efficient progressive technique for noisy gray-scale images, and apply it to document processing. The new technique will preserve the topological properties of the segmented information that will be presented for further processing and recognition processes.

In pursuing this line, we used for our experiments real life Zip code images from the data base provided by the CEDAR at SUNY Buffalo.

831 images were used to train the system, and another 814 to test and validate its performance. The training sets are used to select optimal values of scale parameter Dim (mask dimension). The following settings have been adopted for the mentioned application. For the sake of extracting good data quality, for each scale in this approach we first determine the mask size (Dim), and then we compute the standard deviation of the kernel operator, using the formula σ = Dim / 6. The step scale for Dim equals 2 for this application:

i) General case : **VC $\in$ [0.195, 0.49]**

According to section 4.2, we know that when the mask size varies between the values 9 and 15, we will extract good data quality with noise immunity, then:

DimMin = 9 SigmaMin = 9/6 = 1.5
DimMax = 15 SigmaMax = 15/6 = 2.5

ii) If **VC < 0.195** OR **dimX*dimY < 15000** OR **VC*dimX*dimY < 6500**

In this case, the transfer function is needed to enhance the quality of handwritten data. In order to minimize the effect of this operation, we should work with a lower level, hence:

do image enhancement: transfer function
DimMin = 9 SigmaMin = 9/6 = 1.5000
DimMax = 13 SigmaMin = 13/6 = 2.1666

iii) If **VC > 0.49**

This means we are dealing with thick and dark handwritten information. The method should use higher levels in order to extract data:

DimMin = 9 SigmaMin = 9/6 = 1.5000
DimMin = 17 SigmaMin = 17/6 = 2.8333

iv) If **VC > 0.55**

In this particular case:

DimMin = 9 SigmaMin = 9/6 = 1.5000
DimMin = 27 SigmaMin = 27/6 = 4.5000

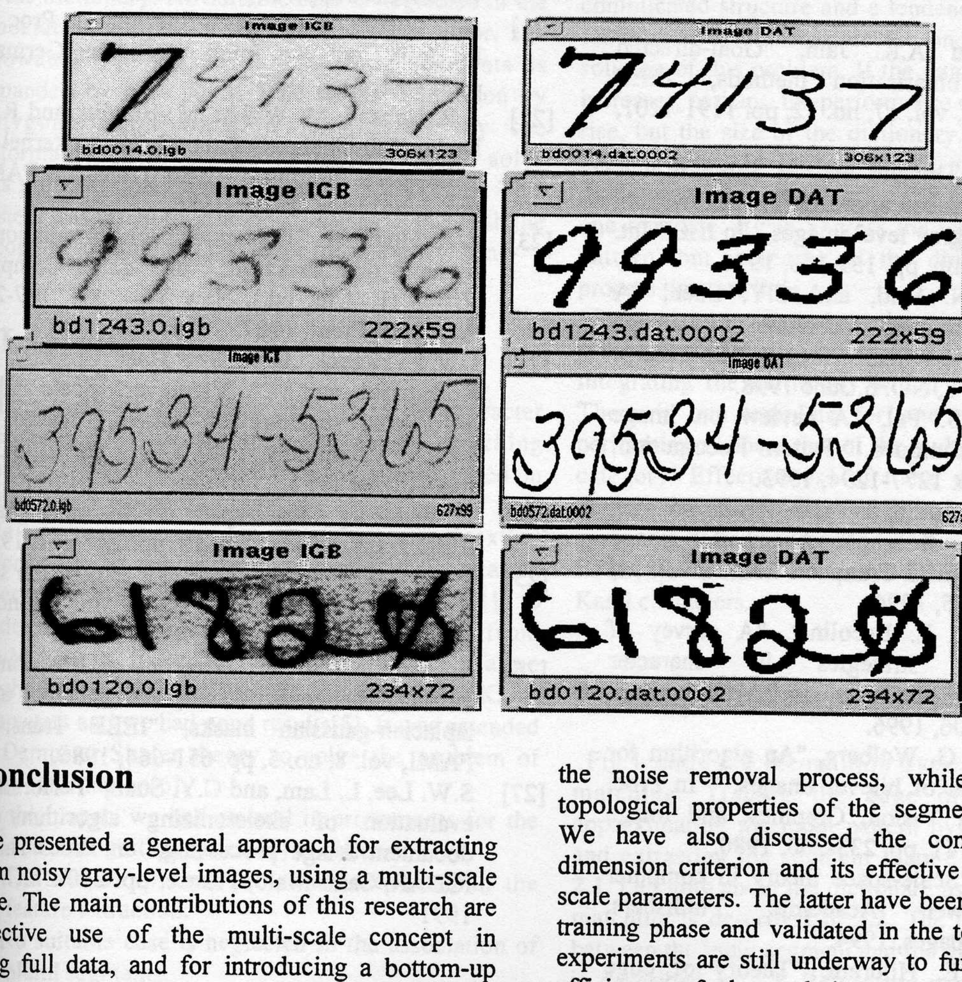
The optimal value of Dim should produce the best performance results, even though performance evaluation of image processing techniques such as image segmentation is a non-trivial task to pursue. In tackling this problem, a set of human visual criteria were defined where each criterion is given a weight. Several performance techniques for image segmentation [4] and thinning [27] follow the same approach. In this application, the performance was analyzed by visual inspection considering criteria such as the thickness, intensity, and connectivity of strokes of character shapes.

Table 1 indicates that training sets of images have good and satisfactory results, which are confirmed by the testing set.

Table 1: Total rate of successful segmentation

	BD 0100 (82 img)	**BD 0200** (88 img)	**BD 0300** (82 img)	**BD 0400** (86 img)	**BD 0500** (80 img)	**Total** (418 img)
	98.78%	98.86%	100.00%	100.00%	98.75%	99.28%
	BD 0600 (87 img)	**BD 0700** (81 img)	**BD 0800** (79 img)	**BD 0900** (82 img)	**BD 1000** (84 img)	**Total** (413 img)
Training set	94.25%	98.76%	96.20%	92.68%	98.81%	96.13%
	BD 1100 (79 img)	**BD 1200** (79 img)	**BD 1300** (84 img)	**BD 1400** (90 img)	**BD 1500** (80 img)	**Total** (412 img)
	94.94%	94.94%	97.62%	96.67%	96.25%	96.12%
	BD 1600 (85 img)	**BD 1700** (71 img)	**BD 1800** (86 img)	**BD 1900** (77 img)	**BD 2000** (83 img)	**Total** (402 img)
Testing set	97.65%	97.18%	98.84%	98.70%	100.00%	98.51%

Some sample images



7. Conclusion

We have presented a general approach for extracting data from noisy gray-level images, using a multi-scale technique. The main contributions of this research are the effective use of the multi-scale concept in extracting full data, and for introducing a bottom-up approach that is more accurate and efficient than the top-down one, in the context of pre-processing Zip Code images. It constitutes an efficient tool to control the noise removal process, while preserving the topological properties of the segmented information. We have also discussed the contribution of the dimension criterion and its effective use for choosing scale parameters. The latter have been tuned during the training phase and validated in the testing phase. The experiments are still underway to further evaluate the efficiency of the technique on a larger data set. Furthermore, since we intend to classify the extracted patterns, we will use the recognition rate as an objective criterion to compare and assess our method with other proposed methods. The experiments have been conducted on the CEDAR data base that containing ZIP Codes scanned in US Postal office from live mail at 300 ppi 8-bit gray level. Very satisfactory segmentation results have been obtained.

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