Camera Calibration with a Viewfinder

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Abstract

To answer the industrial need for simple camera calibration procedure, we propose a new method that requires a simple calibration object composed simply of a box and two crosses. The box is opened in the front where a large cross, made of wires, is attached while another is drawn (or attached) at the bottom. Both crosses are perfectly aligned similarly to a viewfinder. The viewfinder is first oriented with respect to camera such that the optical axis of the camera passes by the center of both crosses, allowing the display of a single (superimposed) cross and an immediate reading of the coordinates of the optical axis. Then, using the similar triangles theorem, the focal distance can be easily estimated. In addition, if necessary, the method can determine the orientation of the CCD matrix if it is not perfectly perpendicular to the optical axis by solving a simple linear system. This method should be particularly useful for calibration of cameras in situ, such as microscopes or embedded cameras.

1 Introduction

Calibration is a heavily worked on area in vision because it is necessary to estimate 3D distance information contained in an image. It allows to model mathematically the relationship between the 3D coordinates of an object in a scene and its 2D coordinates in the image [1,2].

The parameters of the camera are classified in two categories: internal parameters which define the properties of the geometrical optics and the external parameters which define position and orientation of the camera. More specifically, the camera calibration consists in determining the intrinsic (focal length, optical center, scaling factors) and/or the extrinsic (camera rotation and translation) parameters [4,5].

Numerous techniques use an object of calibration of known dimensions for this purpose. This object could be a plane [3,4,6,9,10,13], a cube [1,2] or a sphere with several landmarks (e.g. crosses, circles or squares) on it.

Most methods have in common the same geometrical model, the pinhole model, and sometimes takes into account optical distortions. They typically integrate the size of the pixels since this is typically provided by the CCD matrix manufacturer and assumed accurate [5,7].

The classical method uses equations of linear perspective projection (pinhole model) to extract intrinsic as well as extrinsic parameters. It requires the resolution of a linear system of 2n equations, where $n \geq 6$ is the number of non-coplanar points (landmarks of known 3D positions). Other methods such as calibration from vanishing points and self-calibration are also possible. However these methods are not simple and require several landmark identifications or detections [1,2].

In this paper we suggest a very simple procedure to obtain the camera intrinsic (and extrinsic) parameters that is as easy as aiming through a viewfinder.

2 Background

An ideal camera [3] (pinhole model), with an image plane without distortion and governed by the laws of projective geometry is assumed (figure 1).

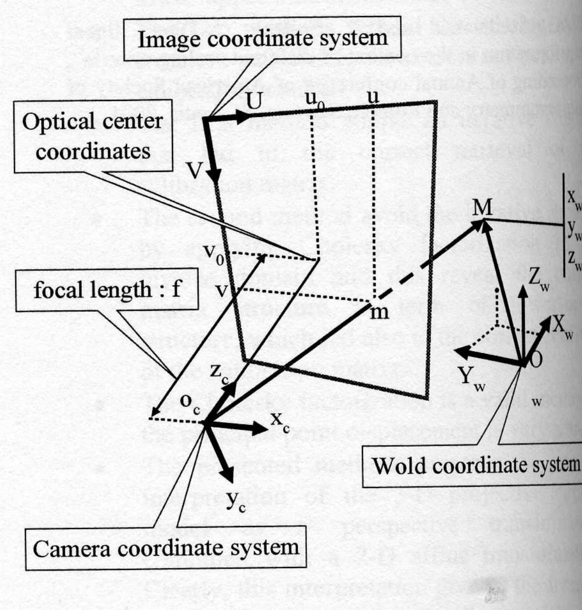

Figure 1 : Perspective projection.

The image plane is situated at a distance *f* (focal distance) of the origin (o_c) (center of projection) of the camera coordinate system. The intersection of the optical axis with the projection plane has coordinates (u_0, v_0) also called optical center of the image.

A point M (see Figure 1) defined by its world coordinates (x_w, y_w, z_w) can be transformed by a translation and / or a rotation in the camera system coordinate (x_c, y_c, z_c) with:

$$\begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{12} & t_x \\ r_{12} & r_{12} & r_{12} & t_y \\ r_{12} & r_{12} & r_{12} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix} \quad (1)$$

where

$r_{11} = \cos\beta\cos\gamma$

$r_{12} = \sin\alpha\sin\beta\cos\gamma - \cos\alpha\sin\gamma$

$r_{13} = \cos\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma$

$r_{21} = \cos\beta\sin\gamma$

$r_{22} = \sin\alpha\sin\beta\sin\gamma + \cos\alpha\cos\gamma$

$r_{23} = \cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma$

$r_{31} = -\sin\beta$

$r_{32} = \sin\alpha\cos\beta$

$r_{33} = \cos\alpha\cos\beta$

This transformation includes the extrinsic parameters: 3 rotations angles and 3 translation components.

Then, to obtain the final image coordinate (***u***,***v***), a perspective projection matrix is added. This step includes the following intrinsic parameters: the focal distance *f*, the center of projection coordinate (u_0, v_0) and the pixel size (p_x, p_y)..

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = \begin{bmatrix} \frac{1}{p_x} & 0 & u_0 \\ 0 & \frac{1}{p_y} & v_0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix} \quad (2)$$

and

$$u = \frac{U}{W} = u_0 + \frac{f}{p_x} \cdot \frac{x_c}{z_c}, \qquad v = \frac{V}{W} = v_0 + \frac{f}{p_y} \cdot \frac{y_c}{z_c} \quad (3)$$

The intrinsic parameters given by the manufacturer are in general just an approximation of the camera parameters (except for the size of pixels), and are incompatible with an exploitable and fine analysis of images. We therefore suggest in the following section a method to obtain these intrinsic parameters (and extrinsic parameters) with a simple calibration object.

3 Calibration object construction

Our technique requires a simple calibration object simply composed of a box and two crosses (figures 2 and 3) similar to a viewfinder. The box is opened in the front and one large cross (in fact a star) is drawn (or attached) at the bottom (defined by the points *C,D,K,L,P,Q,G and H* in figure 2) while another, made of wires, is attached to the front (*A,B,I,J,M,N,E and F*). Both crosses are perfectly aligned. Notice that simpler crosses could also be used (such as *CD,KL* and *AB,IJ* or *EF,MN* and *GH,PQ*) for the calibration object (see figure 2).

3.1 Method

The viewfinder is first oriented with respect to the camera such that the optical axis of the camera passes by the center of both crosses, allowing the display of a "single" cross (superimposition of both crosses, see figure 2). This is similar to aiming with a viewfinder. Since both crosses are aligned and are on parallel planes, an immediate reading of the coordinates of the optical axis (u_0, v_0) is possible at the center of the cross.

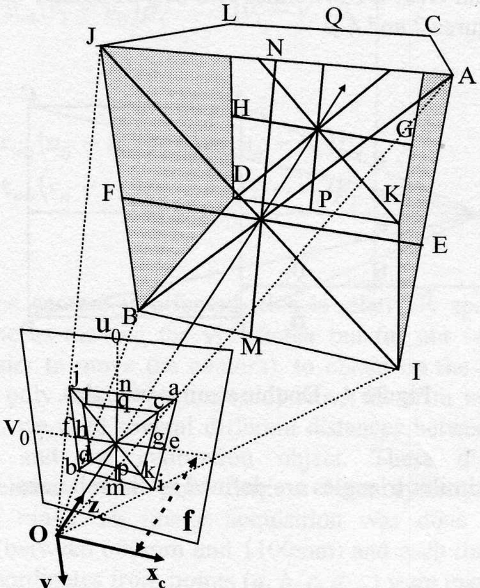

Figure 2 : Projection of the viewfinder on the image plane

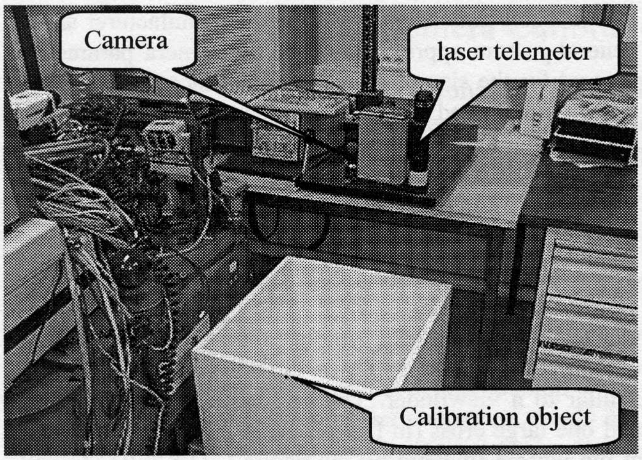

Figure 3 : The calibration setup used in this study.

This is one of the advantages of this method which allows to determine u_0 and v_0 directly without calculation.

The estimation of the remaining unknown intrinsic parameter, the focal length, is obtained by the application of the similar triangles theorem.

As one can see on the figure 2 the perspective projection of the viewfinder on the image plane give us the line segment *ab*, which is the perspective projection of *AB* and *cd* , the perspective projection of *CD* and so on.

One can easily verify that the couple of triangles ***Oab***, ***OAB*** and ***Ocd***, ***OCD*** defines two sets of similar triangles (see figures 2 and 4).

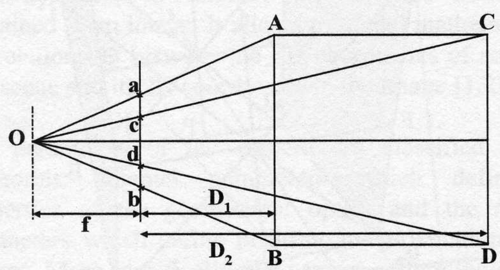

Figure 4 : Double similar triangles.

These similar triangles are defined by the relations:

$$\begin{cases} \dfrac{f}{ab} = \dfrac{f + D_1}{AB} \\ \dfrac{f}{cd} = \dfrac{f + D_2}{CD} \end{cases} \quad (4)$$

where D_1 and D_2 are distances form the image plane to the front and back faces of the viewfinder respectively. Since $D_2 - D_1 = AC = BD = JL = IK$ and $AB = CD$, one can obtain the focal length easily:

$$f = \frac{AC \cdot ab \cdot cd}{AB(ab - cd)} \quad (5)$$

Several combinations of similar triangles can be used but to minimize the reading and computation errors we opted for similar triangles corresponding to the extremities (front and back faces) of the calibration object. We have **8 possibilities** to determine focal length in this way, we can write by circular permutation:

$$\begin{aligned} f_1 &= \frac{AC \cdot ab \cdot cd}{AB(ab - cd)} & f_5 &= \frac{AC \cdot ai \cdot ck}{AI(ai - ck)} \\ f_2 &= \frac{AC \cdot ef \cdot gh}{EF(ef - gh)} & f_6 &= \frac{AC \cdot ib \cdot kd}{IB(ib - kd)} \\ f_3 &= \frac{AC \cdot ij \cdot kl}{IJ(ij - kl)} & f_7 &= \frac{AC \cdot bj \cdot dl}{BJ(bj - dl)} \\ f_4 &= \frac{AC \cdot mn \cdot pq}{MN(mn - pq)} & f_8 &= \frac{AC \cdot ja \cdot lc}{JA(ja - lc)} \end{aligned} \quad (6)$$

One can compute the final focal length as the mean of these values for better accuracy :

$$\bar{f} = \frac{\sum f_i}{n} \quad (7)$$

Notice that in order to add robustness with respect to outliers, one could use the median or a weighted sum instead of the mean.

3.2 Method of weak angles

Our first hypothesis considers that the image (CCD) plane is parallel to the plane (x_c, o_c, y_c) of the camera coordinate system and the front and back faces of the calibration object. In reality, it is usually slightly different, and weak angles may remain. Thus we can re-evaluate equation (1) by including the hypothesis of weak angles. In that case, the sinus of an angle is equal to its angle and the cosine is equal to 1:

$$\begin{bmatrix} x_{ci} \\ y_{ci} \\ z_{ci} \\ 1 \end{bmatrix} \approx \begin{bmatrix} 1 & -\gamma & \beta & 0 \\ \gamma & 1 & -\alpha & 0 \\ -\beta & \alpha & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_{wi} \\ y_{wi} \\ z_{wi} \\ 1 \end{bmatrix} \quad (8)$$

with $t_z = f + D_2$

For a particular point M_i, the perspective projection becomes equal to:

$$u_i \approx u_0 + \frac{f}{p_{xi}} \cdot \frac{x_{wi} - \gamma \cdot y_{wi} + \beta \cdot z_{wi}}{-\beta \cdot x_{wi} + \alpha \cdot y_{wi} + z_{wi} + t_{zi}}$$
$$v_i \approx v_0 + \frac{f}{p_{yi}} \cdot \frac{\gamma \cdot x_{wi} + y_{wi} - \alpha \cdot z_{wi}}{-\beta \cdot x_{wi} + \alpha \cdot y_{wi} + z_{wi} + t_{zi}} \quad (9)$$

We have then obtained a system of **5** unknown parameters (α, β, γ, f *and* t_z) in **2*n*** equations for ***n*** points (in our case ***n*** = **17**). This relation can estimate intrinsic and extrinsic parameters of the camera by resolution with the least square method of the following system:

$$AX = B \quad (10)$$

From this matrix system we obtain the estimation of the focal distance ***f***, as well as orientations α, β and γ around axes x_c, y_c and z_c and t_z by the least square method with:

$$X = (A^T A)^{-1} A^T B \quad (13)$$

4 Experimental results and discussion

A 50 cm x 50 cm x 40 cm box made of wood and open on the top was used to test the approach. We made at the bottom of the box dark crosses (because the inside of the box is white) and we repeated the same thing for the upper face with wires (see figures 2 and 3). Of course, depending of the application, other constructions and sizes are possible. The CCD camera model is TMC-6 from Pulnix (USA) 752x582 (p_x=8.6μm p_y=8.3μm) and the lens is a Computar TV lens 1:1.2 12mm (Japan) while the frame grabber (with low-level image processing capabilities) was an Optibase (Israel). The camera set up enables an adjustment of the six degrees of freedom. To avoid geometric aberrations and chromatic aberrations, we have equipped the camera with an higher quality optical system with aspherical lens. For other optical system, one could minimize the aberrations by reducing the diaphragm aperture and/or by adding a black ring on the periphery of the lens.

$$A = \begin{bmatrix} x_{w1} & 0 & z_{w1} & -y_{w1} & -y_{w1}(u_1-u_0)p_x & x_{w1}(u_1-u_0)p_x & -(u_1-u_0)p_x \\ y_{w1} & -z_{w1} & 0 & x_{w1} & -y_{w1}(v_1-v_0)p_y & x_{w1}(v_1-v_0)p_y & -(v_1-v_0)p_y \\ \vdots & & & & \vdots & & \\ \vdots & & & & \vdots & & \\ x_{wn} & 0 & z_{wn} & -y_{wn} & -y_{wn}(u_n-u_0)p_x & x_{wn}(u_n-u_0)p_x & -(u_n-u_0)p_x \\ y_{wn} & -z_{wn} & 0 & x_{wn} & -y_{wn}(v_n-v_0)p_y & x_{wn}(v_n-v_0)p_y & (v_n-v_0)p_y \end{bmatrix}$$

with

$$B = \begin{bmatrix} z_{w1}(u_1-u_0)p_x \\ z_{w1}(v_1-v_0)p_y \\ \vdots \\ \vdots \\ z_{wn}(u_n-u_0)p_x \\ z_{wn}(v_n-v_0)p_y \end{bmatrix} \quad (11)$$

and

$$X = [f \quad f\alpha \quad f\beta \quad f\gamma \quad \alpha \quad \beta \quad t_z]^T \quad (12)$$

First, the camera is oriented (this is relatively speaking the same as moving the viewfinder but for our setup it was easier to move the camera), to obtain on the image screen, only one (superimposed) cross. Then to test the method, we used several different distances between the camera and the calibration object. These distance measurements were done with a laser system. In a 500mm range, an image acquisition was done every 10mm (between 600mm and 1100mm) and each time the pixel coordinates from points (***a***, ***b***, ***c***, ***d***...) were manually collected. Recall that these points are projections from points (***A***, ***B***, ***C***, ***D***...) of the object calibration (figure 2). With the coordinates collected, we have estimated manually the length of the different segments corresponding to possible combinations of reference points on the calibration object that are projected on the image plane. These pixel data are then converted into metric (mm) data, using pixel size provided by the manufacturer ($p_x = 8.6\mu m$, $p_y = 8.3\mu m$).

For the first method (similar triangles) we suppose that the image plane is parallel with the front and back faces of the calibration box. For various distances, we read the coordinates of the optical center (u_0, v_0) in pixels and we calculate the focal distance f by the similar triangles approach. Then the average of 8 calculations is done to compute the focal distance. Figure 5 shows graphically the different focal distances in millimeter evaluated as a function of the camera-box distance in millimeters. Table 1 summarizes the results.

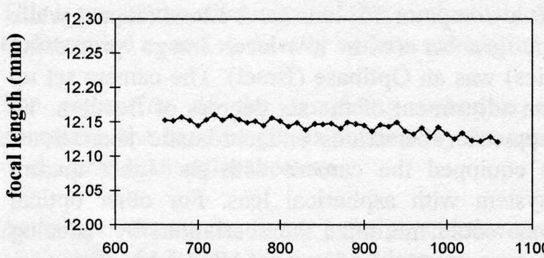

Figure 5 : Focal Length vs. Camera Distance.

f (mm)	u_0 (pixel)	v_0 (pixel)
12.1432	344	288
σ = 0.010578	σ =0	σ = 0

Table 1 Camera intrinsic parameters

With the second method, we modify the projection matrix to include possible weak angles of orientation. Solving the matrix system we obtain very similar results presented in Table 2.

f (mm)	u_0 (pixel)	v_0 (pixel)
12.1582	344	288
σ = 0.01054	σ =0	σ =0

Table 2 Camera intrinsic parameters

Notice that the weak angles α , β and γ were found to be negligible in this experiment.

Figure 6 shows that the other extrinsic parameter t_z follows linearly the camera distance measured with the laser as expected.

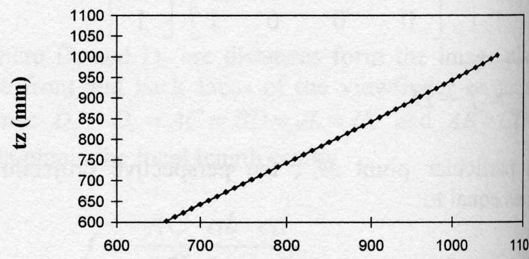

Figure 6: t_z vs. camera distance.

The limit of utilization of our calibration object is linked to equation (5). While *AB* and *AC* are typically measured with a good millimeter tolerance, on the other hand *ac, cd* (representing the projection of the segment *AB, CD*) are measured with a pixel tolerance (although subpixel measurement is possible). Moreover, their difference could also be small and prone to errors as the camera is farther form the calibration box. This can be observed in figure 5 where the variability of the measurements increases with distance. Indeed, the standard deviation of f in the range from 660 to 750 mm is 0.00453 and increases to 0.00774 in the range 950 to 1060 mm. This can maybe also explain the slight decrease of the computed focal distance when the camera is moved away from the calibration object.

In fact, to realize a good calibration, it is important that the object of calibration fills a region between 50 % and 80 % of the size of the image. Below 50 % it is difficult to extract good edges and an error of 1 or 2 pixels on the edge detection can generate significant errors. Above 80 % there is a risk to enter in the distortion domain. This remark is valid for any method of calibration using an object calibration.

For the determination of the optical center we believe that our method is more fine than classical methods because it does not necessitate any calculation since u_0 and v_0 are available directly form the image (center of the superimposed crosses).

For the same camera we made several calibration with 5 calibration viewfinders of various sizes and different shapes (regular forms). The results were all very similar and confirm the good reproducibility of focal length measurement with the proposed method. This method was also successfully used to calibrate a camera used for 3D localization of polygonal shapes by monocular vision.

Notice that ideally the shape of the calibration viewfinder should be chosen by taking into account equation 5 (and 6) to reduce as much as possible the impact of measurement errors (see previous comments).

Finally, results obtained from three other (much more complex) methods are displayed in Table 3 for comparison purposes. One can observe that our parameters are within these values except for v_0 that is slightly lower. Considering the variability of these measurements it seems reasonable to say that our simpler method performs as well as more complex ones at least in this study.

Parameters	Tsaï [3]	Toscani [4]	Multistep[9,11]
u_0	321	350	362
v_0	281	281	284
f	11.21	12.02	12.63

Table 3 Results for three other methods.

5 Conclusion

In this paper we have presented a complementary and alternative method to classical camera calibration. Using a simple calibration object similar to a aiming viewfinder, we have shown that one can directly obtain the image center (u_0, v_0) and with a few measurements the focal distance. Notice that the similar triangles method does not require the computation of extrinsic parameters (camera rotation and translation) to get the intrinsic parameters. The weak angles method can also take into account a CCD matrix plane that is not perfectly perpendicular to the optical axis.

The main advantages of this calibration procedure are its simplicity, its easy calculations and the immediate reading of the optical center. It could also be particularly useful to calibrate cameras in their environment, as for example microscopes equipped with camera or embedded cameras using a viewfinder with the appropriate dimensions and constructions. For instance, with microscope, one could calibrate the complete camera-microscope system by focusing on a (very) small calibration viewfinder located entirely in the depth of field of the system.

Although the alignment of the optical axis in laboratory requires effectively some fine tuning mechanics, this type of procedure should not be a problem in an industrial environment.

6 Bibliography

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